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WAVES AND ELECTRONS

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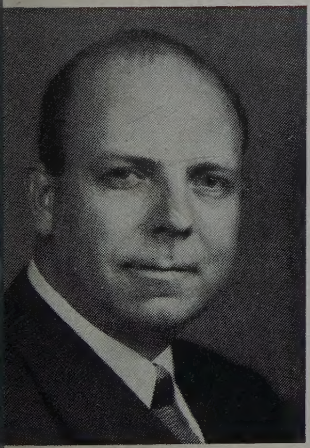

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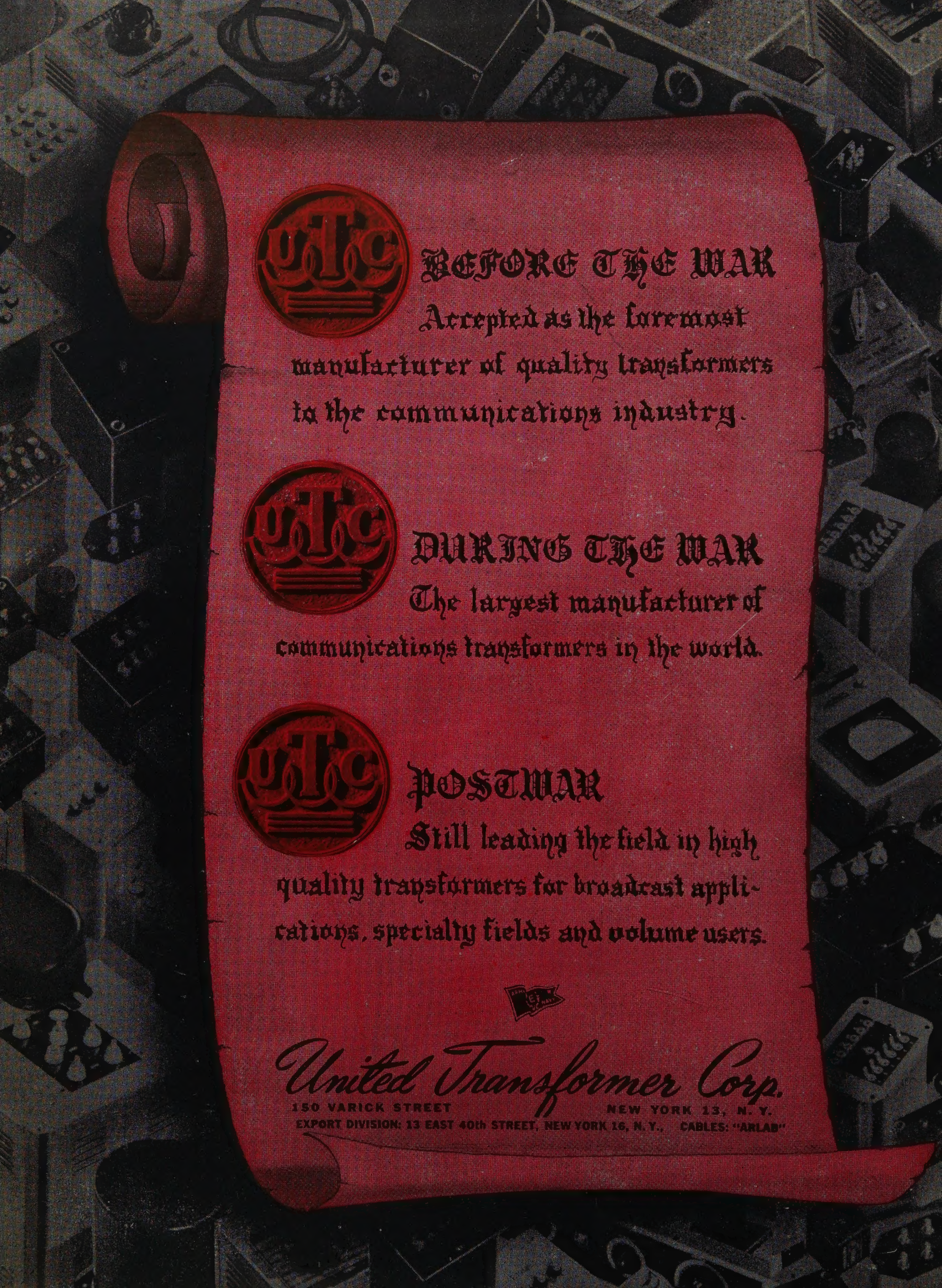
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1946

F. B. LLEWELLYN, PRESIDENT-ELECT—1946

NOT surprising is the growth of the Institute of Radio Engineers during the past few years. The part played by radio and electronics during the war is, in itself, sufficient explanation. More important than the effect of numbers, however, is the increased attention by the public at large to the accomplishments and also to the opinions of engineers. This is a situation that places a very direct and important responsibility both upon the engineer as an individual and upon the technical and professional societies through which his viewpoints are expressed.

Two doors are before us. The one discloses a future where the engineers are plodding along, solving the technical problems as they appear, but taking no part in their broad impact upon the civic and economic factors of the day. In some ways it is a comfortable setting, but not an inspiring one, and the part of the engineer is little more than that of a more or less intelligent robot.

The other door opens on a much wider horizon, but in this case the engineers are assuming their whole responsibilities, as true Sons of Martha, not only in dealing with the strictly technical aspects of their problems, but also in guiding the destiny and application of their work. It is by no means a cloistered environment, but one where commercial, industrial, and economic factors are prominent in the scene. The engineer is required to deal with intangibles and with human nature, where the problems are much more difficult to solve than when confined to inanimate matter.

Yet is there any question which door we should enter? By training and temperament, engineers are capable of providing leadership in broad fields and they should recognize this responsibility. Engineers and scientists who, together, formulate our technical advances, must and should participate actively in their subsequent use and application. It is the duty of every engineering organization to promote that participation by every means available.

Among the plans in this direction that have been formulated during the past few years is a proposed reorganization of our Board of Directors in a way to obtain a broader representation and to shift its administrative functions to the Executive Committee and the Executive Secretary, clearing the way for a more comprehensive consideration of issues and general policy.

Another departure in the same direction is the inclusion of "WAVES AND ELECTRONS" in our publications. This is expected to provide an expanded medium for the expression of engineering opinion on the social and civic applications of technical progress and on the function of engineers in carrying out those applications.

Of major importance also, are the plans for promoting the interchange of ideas among specialists within restricted fields, between specialists in different fields, and finally between different engineering groups and Societies. Under these categories come the sponsorship of specialized conferences by the Technical Committees, the formation of a Speakers' Bureau at the Headquarters office, and the promotion of co-operation with other engineering Societies, both in the Sections and at Headquarters.

These and other means are needed to carry the program ahead, and the active participation of every engineer is necessary for its accomplishment. With past performance as an incentive, can there be any doubt of the future? I wish that every engineer who can participate in Institute activities would write to me or to the Institute office, with a statement of his special inclinations.

A New Method of Amplifying with High Efficiency a Carrier Wave Modulated in Amplitude by a Voice Wave*

SIDNEY T. FISHER†, SENIOR MEMBER, I.R.E.

Summary—**This paper describes a new high-efficiency amplifier circuit with a quantitative analysis of its operation. This circuit operates by dividing the wave in several sections, amplifying each section separately and recombining the sections in the output to produce a larger wave of the original form. The circuit has special application to controlled-carrier systems and relatively high efficiencies are obtained, the comparison with conventional arrangements being most favorable at low modulation levels.**

Introduction

THE PROBLEM of a high-efficiency linear amplifier is one with which radio engineers have been concerned for 25 years. Power amplifiers for either unmodulated or frequency-modulated carrier waves operate with such high efficiency, of the order of 75 per cent, that no considerable improvement is necessary. A power amplifier for a carrier wave modulated in amplitude by a speech wave still presents an outstanding problem, and it is the purpose of this paper to develop a general line of attack on the problem.

Several solutions have previously been offered. The most wide-spread arrangement in use today is the class C radio amplifier modulated at high level by a class B audio amplifier. Other more complex arrangements in less common use are the Chireix "out-phasing modulation" method, and the Doherty high-efficiency circuit.

The Linear-Amplifier Problem

When a wave containing amplitude modulation is to be amplified, the amplification must be closely linear. A conventional class B amplifier is linear, and when operated continuously at its maximum capacity, such an amplifier will have an efficiency of the order of 66 per cent. An amplitude-modulated wave has a value on peaks of modulation of twice the carrier value, so that a class B linear amplifier transmitting such a wave must have a maximum capacity twice that of the carrier wave unmodulated. Since, over any considerable period of time a voice modulating wave has a very low average value, we are not far wrong in considering the efficiency of the circuit for the carrier wave alone as its actual performance. The efficiency of the class B amplifier is about proportional to the root-mean-square value of the wave being transmitted by it, so that we have in conventional linear amplifiers intended for transmitting a carrier wave amplitude-modulated by a speech wave, and where the modulation may reach 100 per cent, an average efficiency of only about 33 per cent. This means that, for every watt delivered to the antenna, about two watts of power is dissipated as heat at the anodes of the output tubes.

Two disadvantages are immediately apparent: first, the cost and difficulty of providing this relatively large amount of direct-current power at high voltage are considerable; and second, unduly large power-amplifier tubes must be employed in order safely to dispose of this amount of heat.

Aside from the question of efficiency of the power amplifier for the conventional transmission method, another factor should be considered. In an amplitude-modulated system, three major components are contained in the output wave; the carrier wave and two sideband waves. The two sidebands are equal in power, and together contain one half of the power contained in the carrier wave, for continuous maximum modulation. In other words, a radio transmitter with an unmodulated carrier of 100 watts, at maximum modulation transmits 150 watts, of which 25 watts is contained in the lower sideband, and 25 watts in the upper sideband. Telephonic speech may be assumed to have an average value of about 20 per cent of the maximum value over any considerable period of time. The total power in the two sidebands is, therefore, for average telephonic speech, not one half the power in the unmodulated carrier, but is given by the following expression:

$$\frac{\text{Sideband power for telephonic speech}}{\text{Unmodulated carrier power}} = \frac{1}{2} \times \frac{1}{5^2} = \frac{1}{50}.$$

This means then that, for the 100-watt unmodulated carrier, the average sideband power over some period would be about 2 watts.

Since the intelligence is wholly contained in the sidebands and not in the carrier, the real efficiency of such a system is seen to be surprisingly low. Suppressed-carrier and controlled-carrier systems have been suggested and put to use in a wide variety of applications but have heretofore suffered from a rather fundamental disadvantage. If, for example, the amplitude of the carrier wave in a radio transmitter is adjusted by a circuit operating from the envelope of the audio modulating wave, so that for all values of the modulating wave the percentage modulation is kept close to 100 per cent, then in the conventional class B linear amplifier, which is required to raise this wave to a high power level, the efficiency will not be 33 per cent. Actually, it will be

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very much less, because the average value of such a wave will be much less over a period of time than the average value of the wave in a conventional system, which is very close to the unmodulated carrier wave. If we again assume the average value of the telephonic speech wave is about 20 per cent of the peak value, then in such a system we will have a carrier wave which has an average value of 20 per cent of the maximum carrier which the system will transmit, and this wave will therefore have 1/25 of the power of the maximum carrier rating.

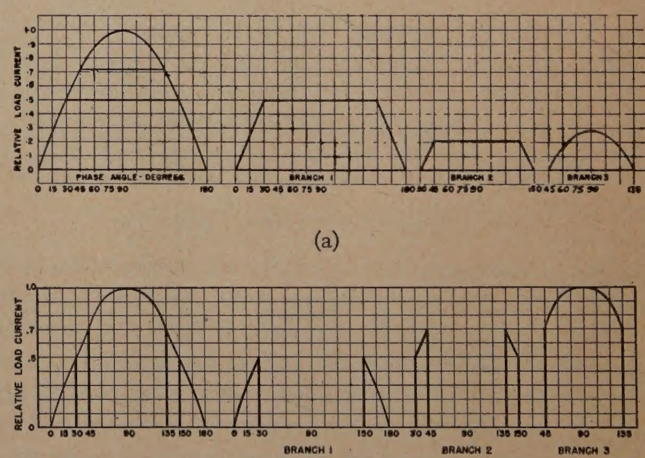
Taking the case of the transmitter with 100-watt unmodulated carrier, the control of the carrier wave to maintain substantially complete modulation for all values of the audio modulating wave will not affect the peak rating which the power-amplifier stage must have, and we incur the serious disadvantage that this output stage will operate at an average power level of about 1/25 of its peak power rating. The over-all efficiency will therefore be very low (of the order of a few per cent) and the apparent advantage obtained by a controlled-carrier system has largely been offset by the low efficiency of the linear power amplifier which must be employed.

In the high-efficiency circuit of Doherty, linear amplification is obtained at an efficiency of around 60 per cent, which is very nearly twice the efficiency of the conventional class B modulated-wave amplifier. This efficiency of 60 per cent is the same order of efficiency as is achieved by a class C carrier amplifier modulated at high level by a class B audio amplifier, and the choice between the two systems lies in the practical details of components, ease of adjustment, and first cost, which for any individual application may be quite different for the two approaches to the problem. We have previously noted that, based on average speech, the sideband power is only about 2 per cent of the unmodulated carrier power, so that if we take the ratio of sideband power to direct-current input to the power-amplifier stage (which is actually a true statement of the utility of the conversion which we obtain in the power-amplifier stage of a radio transmitter) then it is seen that the true efficiency lies between 1 and 2 per cent, and we are back to the same order of efficiency as is obtained in a controlled-carrier system using a conventional class B linear amplifier. A controlled-carrier system using a Doherty or Chireix amplifier will have better efficiency than this, although the efficiency will still be in the region below 10 per cent, and for many applications where tuning over a frequency band is required, the complexity of these circuits is prohibitive.

Proposed Circuit

On consideration of these facts, it is realized that this problem, one of the most important in all radio engineering because of the large use of radio transmitters for aircraft and other mobile uses where weight, size, tube cost, and power requirements are serious considerations, requires a completely new attack. It is believed that the proposal which follows is a basically correct approach to the problem.

The anode dissipation in a vacuum tube goes to a low value when either the anode current is reduced to a low value without exceeding the allowable anode voltage, or the anode voltage is reduced to a low value without exceeding the maximum anode current. It will be obvious that the wave form which fulfills both these conditions will be transmitted with maximum efficiency, and that this wave form is a square-topped wave. In such a wave, the energy is completely contained in rectangular pulses which rise instantaneously from zero to the maximum value, and drop to zero from this maximum value instantaneously. The energy is therefore transmitted wholly during the time at which the maximum allowable anode current is being transmitted, and under this condition the ratio of voltage drop across the load to voltage drop across the tube is a maximum. If signaling systems were called upon to transmit only such wave forms, of constant amplitude, we would then have linear power amplifiers which would operate with an efficiency of the order of 90 per cent, using conventional tubes. It appears that a successful approach to the actual problem can be developed from this simple statement.



(a)

(b)

Fig. 1—Division of sinusoidal wave into sections, each of which can be amplified with higher efficiency than the original wave.

The solution to the problem resolves itself into changing the actual signal wave to the form in which it can be transmitted with the highest efficiency; that is, having it approach a square-topped wave as nearly as possible. Fig. 1 shows two ways in which this can be done. The wave can be divided into a number of sections, horizontally (Fig. 1(a)) or vertically (Fig. 1(b)), that is, on either an amplitude basis or a time basis. The method to be proposed therefore consists of the following steps: (1) Subdivision of the wave into components that approximate the optimum, rectangular wave form; (2) Amplification of these components separately in vacuum-

tube amplifiers; (3) Recombination of the separate components in the output circuit of the amplifiers so as to reproduce the original wave at a higher power. That is to say, high-efficiency amplification is achieved by dividing the wave on an amplitude basis into several sections, in practical cases, about three; transmitting each of the sections through a power amplifier whose peak allowable current is that of the section being transmitted, and then combining the three sections at the output by connecting the three amplifier branches to a common load circuit so that the original wave form is again obtained. This arrangement results in some circuit complexity, but increases by a large factor the plate-circuit efficiency of the amplifier. It also reduces the required tube complement, with a corresponding reduction in weight and size of the associated apparatus, since the increase in efficiency chiefly manifests itself in a reduction of the amount of power dissipated at the anodes of the power amplifiers.

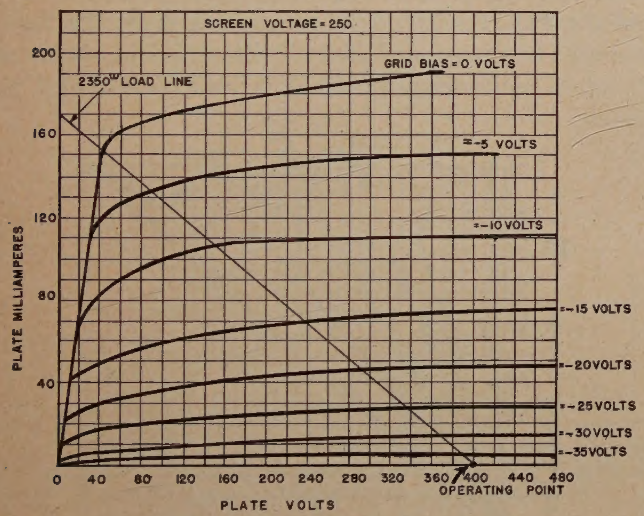


Fig. 2—Plate-current—plate-voltage characteristic of typical small beam tetrode, showing low voltage drop across tube at high values of plate current.

Circuit Operation

An approximate way of regarding the system proposed is to think of it as a series of class C amplifiers, whose inputs are driven by different sections of the wave, the sections being selected by a combination of grid bias and grid drive in an arrangement which can be termed an "amplitude filter." Each branch then amplifies the section of the wave which it receives with higher efficiency and higher output than that with which it could handle the whole wave, and the sections of the group are combined in the output circuit so that a linear relation is obtained between input and output.

A reference to Fig. 2 demonstrates in a qualitative way the major point involved. This illustration shows the plate characteristic, and Fig. 3 shows the grid-voltage—plate-current characteristic of a typical small transmitting tube. When the tube is operated as a class B amplifier into the rated load impedance, the plate current rises along the load line to a maximum value which is determined by the allowable anode heating and the allowable cathode current. It will be noted for the tube whose characteristics are given, that when the grid is driven to zero voltage, the maximum operating point for this tube, 10 per cent of the plate voltage appears across the tube, and 90 per cent across the load. That is to say, at this point the tube is transmitting

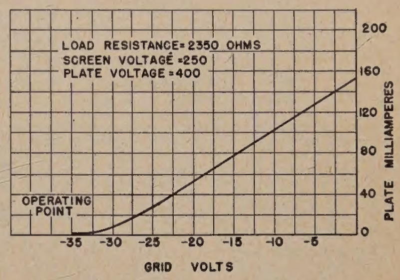


Fig. 3—Grid-voltage—plate-current characteristic of the tube of Fig. 2.

power at an instantaneous efficiency of 90 per cent. However, when the tube transmits a sine wave or a signal-modulated wave, only a small part of the energy is transmitted at, or near, this high-efficiency point, and most of the energy content of the wave is transmitted at much lower efficiencies; a sine wave being transmitted with about 60 per cent, and a modulated wave with about 35 per cent over-all efficiency. It will be apparent that this tube would transmit a square-topped wave with an efficiency of 90 per cent, so that for a given plate dissipation the tube would have a power output for the square-topped wave about four times greater than for the sine wave, and about six and one-half times greater than for the carrier wave 100 per cent modulated by a signal.

This leads to the present proposal of dividing the sine wave or modulated wave into a series of pulses, each of which has a form more nearly approaching the required

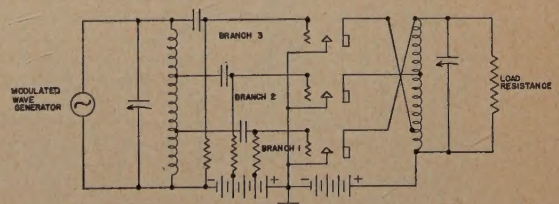


Fig. 4—Schematic circuit of a three-branch linear amplifier, with the grid drive and grid bias individually adjusted for each branch.

rectangular form, amplifying these pulses through separate power amplifiers whose peak allowable currents are the same as the maximum value of the pulses, and then recombining the pulses in a common load circuit to form the original wave form.

In practice this rather roundabout method has been found to work out with surprising ease. Fig. 4 shows an outline circuit of a three-branch modulated-wave linear

amplifier. Each of the three branches has its grid drive and grid bias individually adjusted so that the branches transmit current in sequence and not simultaneously. Branch 1 is biased at cutoff, so that it operates as a conventional class B amplifier. It receives the lowest grid drive. Branch 2 is biased beyond cutoff, and it has

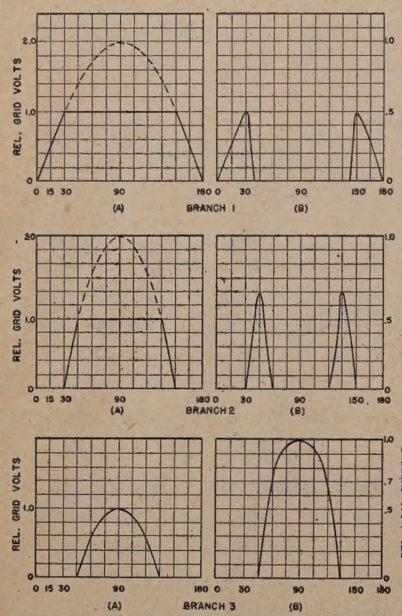


Fig. 5—Division of input wave through a three-branch amplifier.

a greater grid drive. Branch 3 is biased to about twice cutoff, and it has the highest grid drive. These three branches are connected to the load through a common plate coil, and their load impedances are adjusted about

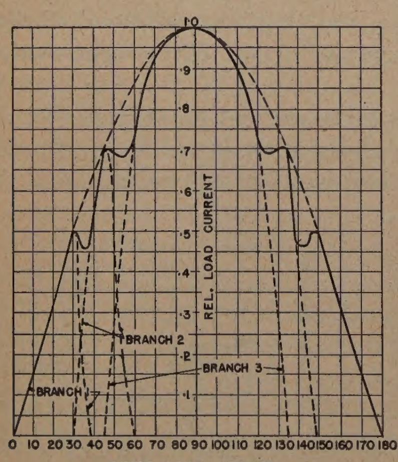


Fig. 6—Recombination of the wave sections of Fig. 5 in the output circuit of a three-branch amplifier.

in inverse proportion to the grid drive. Branch 1 has the highest load impedance, branch 2 an intermediate load impedance, and branch 3 a load impedance about one half that of branch 1.

The operation of this circuit can now be described with reference to Figs. 5 and 6. As the wave commences, branch 1 immediately starts to draw plate current, since it is biased to class B operation. As the wave advances it reaches a point, shown as a relative grid voltage of 1.0 and a relative load current of 0.5, where the peak allowable current of branch 1 tube is reached. At this point, the grid commences to draw current and biases itself back due to the direct voltage set up across the grid leak. At the same time the plate current of branch 1 decreases abruptly because at this point in the wave branch 2 has started to draw plate current and is delivering power to the load from a higher voltage source than does branch 1. Similarly, as the wave advances, branch 3 draws plate current and branch 2 at this point has its plate current abruptly reduced. The same process takes place in a reverse order when the wave has passed its maximum value and decreases again to zero. It will be seen that each tube operates linearly over a range of amplitude for which it delivers power, and nonlinearly outside this range. The three groups of pulses are delivered in sequence to the load resistance, and the way in which they combine is shown in Fig. 6. It might be mentioned that the illustrations shown are copies of oscilloscope patterns in an experimental amplifier.

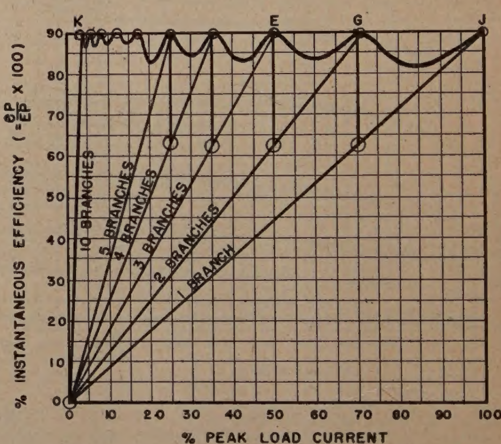


Fig. 7—Per cent instantaneous efficiency plotted against per cent peak load current, for the transmission of a sine wave through amplifiers of different numbers of branches.

It will be noticed that the recombined wave has, for a three-branch arrangement, an appreciable content of the ninth harmonic. Other distortion products are almost entirely lacking. In any radio-frequency application it is, of course, rather easy to reduce the ninth harmonic by any factor desired; and where this circuit is used for audio frequencies, the harmonic content can be reduced about as desired by the application of negative feedback.

Efficiencies Obtained in New Circuit

It will be apparent that the efficiency of this circuit is high, even for low values of the wave being transmitted, since the instantaneous efficiency rises to about 90 per cent as the maximum current in each branch is reached. This is shown in Fig. 7, which is a plot of per

cent instantaneous efficiency against per cent peak load current for the transmission of a sine wave. For the conventional class B amplifier the efficiency is assumed to be proportional to the peak load current, rising to a value of about 90 per cent at 100 per cent of the allowable current. The plot shows how this efficiency curve varies as circuits of varying numbers of branches are used. In each case for which the data is given on this figure, the power ratio in successive branches is 2 to 1; that is, a 3-decibel separation. For a three-branch circuit, for example, the case illustrated by the previous wave-form curves, the instantaneous efficiency rises to 90 per cent at 50 per cent of the maximum load current, and the efficiency does not depart far from this value right up to the maximum power from the over-all circuit. For waves of low amplitude, the efficiency of this circuit is therefore quite good, and in fact is the same for waves of half the maximum amplitude as the efficiency of a conventional class B amplifier for waves of maximum amplitude. If as many as ten branches are used, then the efficiency of the

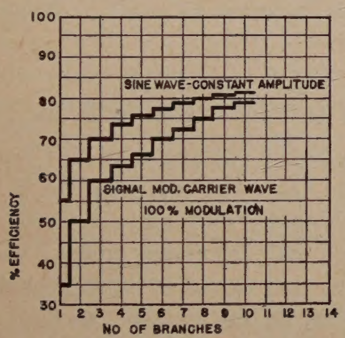


Fig. 8—Per cent efficiency of transmission of a sine wave and of a carrier wave 100 per cent modulated by a sine wave, through amplifiers of different numbers of branches. In each case, succeeding branches have a 3-decibel difference in output power.

circuit for waves of 4.4 per cent of the maximum amplitude is the same as the efficiency of the conventional circuit for waves of the maximum amplitude, and for waves of amplitude higher than 4.4 per cent, the efficiency steadily improves to a value in excess of 80 per cent for waves of the maximum amplitude.

Fig. 7, which was obtained experimentally, is further explained by Fig. 8, which shows the efficiency obtained in amplifiers of different numbers of branches in which 3-decibel separation exists between the branches. The efficiencies are shown both for a sine wave of constant amplitude, and for a signal-modulated carrier wave with 100 per cent modulation, and were derived experimentally using the tube whose characteristics are shown in Figs. 2 and 3. Based on this data, it appears that, for normal applications involving speech modulation, an arrangement of about three or four branches gives the practical compromise between efficiency and circuit complexity in the case of either type of wave. It is likely that for amplification of audio frequencies the greater complexity of the circuit, because of the impossibility of tuning the load, would dictate a smaller number of branches, either two or three. In the audio-frequency case, the "amplitude-filter" arrangement is not so readily obtained by the adjustment of grid drive and grid bias of the power amplifiers, and for such applications it will occasionally be necessary to have separate signal-shaping driver stages, each power-ampli-

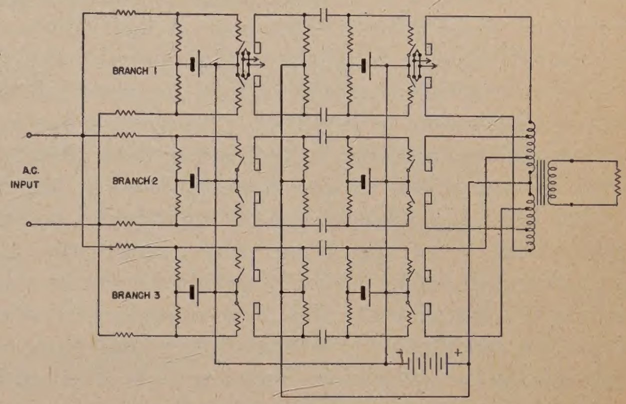


Fig. 9—Schematic circuit of a three-branch audio-frequency amplifier, with separate "signal-shaping" stages.

fier stage operating as a conventional class B amplifier. Such a circuit for a push-pull three-branch audio-frequency amplifier is shown in Fig. 9. In this case the division of the signal into sections is accomplished by small driver tubes which accomplish their function by individual adjustments of the grid drive, grid bias, and plate load.

Application to Controlled-Carrier Systems

It is apparent that the linear-amplifier system described, whose efficiency remains relatively high for low amplitudes of the transmitted wave, has special advantages to offer as a power amplifier for a signal-modulated wave in which the carrier is either controlled

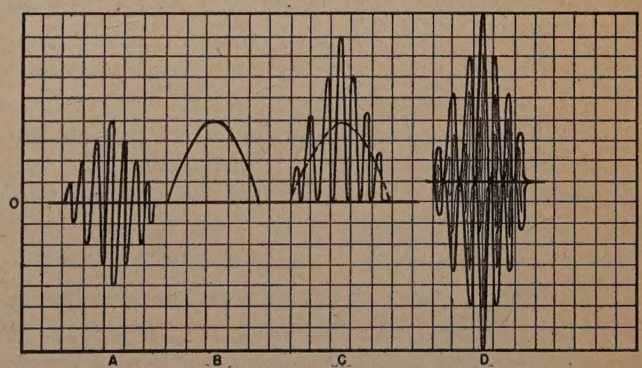


Fig. 10—Wave forms obtained in the operation of a controlled-carrier modulator.

so as to keep the per cent modulation substantially constant and high, or in which the carrier is suppressed. Since suppressed-carrier systems are of rather a special nature and require special receivers, consideration of a controlled-carrier system is of more interest in the present application. In a controlled-carrier system, in general

it is not necessary to use special receivers, and the transmission is essentially no different from constant-carrier systems.

The usual way in which a controlled-carrier system operates is to derive from the audio-frequency modulating wave a unidirectional pulse which has the form of the envelope of the audio-frequency wave. Thus in Fig. 10, the input wave *A* is rectified and filtered to provide the unidirectional pulse *B*. This pulse is then added to the original wave to form the unidirectional wave *C*. It is this wave which is introduced into the system ahead of the modulated-wave amplifier. This wave is applied to the modulated amplifier in such a way that, when no speech current exists, carrier is transmitted at only a very low level, say 5 per cent of the maximum capacity of the system. When speech current is applied, the carrier is increased proportionately to this current so that the output wave consists of a voice-modulated carrier wave whose modulation is substantially complete for all amplitudes of the voice wave. Such a wave is shown as *D* in Fig. 10. Since average voice modulation is only about 20 per cent, it is seen that the range of amplitudes of the output wave will vary, not in a ratio of 2 to 1 as in the conventional transmitter, but in a ratio of, say, 30 to 1, with the average amplitude about one fifth of the peak amplitude instead of about one half the peak amplitude as is the case in a conventional system. It will be recognized that these factors are responsible for the low efficiency of conventional linear amplifiers in controlled-carrier systems, and that the ability of the circuit outlined in the previous paragraphs to sustain its efficiency at low amplitude will be of great value for this type of transmission. For example, to consider again a transmitter with an unmodulated output of 100 watts, if this carrier is so controlled that it drops to, say, 5 watts in the absence of modulation, it will rise to a total value of 150 watts, averaged over an audio-frequency cycle, for 100 per cent modulation. Considering average modulation as 20 per cent, the average power content of the carrier plus the sidebands will be somewhat more than 6 watts. This value would be 6 watts if the carrier were completely suppressed during silent periods, but the constant carrier output of 5 watts combined with the modulation gives an average carrier plus sideband output of about 10 watts. This output power will be generated in a three-branch amplifier with an efficiency of around 30 per cent. This can be determined by applying the data of Fig. 7 to the wave form *D* of Fig. 10. That is to say, the direct-current input to this output stage will be about 33 watts for telephonic speech. This compares with the case of the conventional class B amplifier operating on a controlled-carrier system where, under similar conditions, the direct-current input is of the order of 200 watts, or of the class C carrier-amplifier modulated by the class B audio amplifier using a constant carrier where the direct-current input is about 300 watts.

Conclusions

On a basis of direct-current power input, this arrangement therefore appears to have an advantage of at least 5 to 1 over circuits now in use, and in some cases this improvement might be 10 to 1. It is possible that these advantages will not in all cases be obtained in practical apparatus due to the greater circuit complexity, but substantially the improvement to be expected should be obtained. A further point is that with this circuit the total power dissipation of the output stage is now considerably reduced. For instance, in the example cited above, the power to be dissipated by the anodes of the power-amplifier tubes for average telephonic speech is about 23 watts. The power to be dissipated by the anodes of the power amplifier in a controlled-carrier system, using a conventional class B power stage is about 190 watts, and in an output stage in which a class C amplifier is plate modulated by a class B audio amplifier, it is about the same. The anode dissipation in the system described is therefore only about 15 per cent of that obtained in conventional systems, and the tube complement employed is of correspondingly smaller capacity.

Practical Variations of Basic Circuit

A large number of variations of this basic idea is obviously possible. Only the "vertical" division of the wave has been considered. "Horizontal" division is also possible, in which all branches may transmit current simultaneously; this would involve a bridge or hybrid-coil arrangement in the output, so that the branches could supply current simultaneously to the load, without mutual coupling. Both series and shunt plate-supply arrangements should be considered. In place of the "signal-shaping" arrangement employing adjustments of grid drive and grid bias to set up the sections of the wave, the plate current of one branch can be utilized to "trigger" the grid bias of the succeeding branch. By using a divided direct-current power supply, the branches can be arranged in parallel or in series to deliver power to a single load impedance, instead of the divided load described. The adaptations of this circuit to a modulated amplifier and to an oscillator are straightforward.

Special tubes, having a higher ratio of plate current to plate dissipation than those currently used, will have particular value in this circuit. New forms of tubes, employing multiple grids or multiple anodes, with heat interchange between the anodes, appear to have useful possibilities.

The foregoing material gives a qualitative description of the operation of the new circuit as a linear amplifier. A thoroughgoing theoretical analysis is hardly justified at this stage in the development of the art, since the textbooks provide an adequate background for this and most other configurations of the familiar circuit elements. Following the text is a series of appendixes,

written by E. S. Kelsey, which presents an analytical treatment of this circuit, together with comparisons of it with conventional and other high-efficiency circuits.

Acknowledgment

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Appendix I

Efficiency of an Amplifier for Signals of Arbitrary Wave Form

In this appendix, formulas will be derived for the theoretical efficiency of an amplifier when the signal is of arbitrary wave form. Two conditions of plate supply require consideration; namely, constant-voltage series feed, and constant-current parallel feed. By applying the principle of duality, it will not be necessary to carry the analysis through for both cases, as the two circuits are duals and the equations for the two cases are similar, with currents and voltages interchanged.

Instantaneous Efficiency

With series feed, let E_B be the direct-current power-supply voltage, e_P the plate voltage, and e_T the voltage at the load terminals. Then

$$E_B = e_P + e_T.$$

Let the plate current be i_P; if the load is connected directly in the plate circuit, this will equal the load current and the instantaneous power input and ouput will be

$$p_1 = E_B i_P$$

$$p_2 = e_T i_P$$

giving for the instantaneous efficiency

$$\frac{p_2}{p_1} = \frac{e_T}{E_B}. \tag{1}$$

If a transformer is used between the plate circuit and the load with a transformation ratio

$$n = e_T/e_L$$

the efficiency, neglecting transformer loss, will be

$$\frac{p_2}{p_1} = \frac{ne_L}{E_B}. \tag{2}$$

Efficiency for Complete Signal Wave

If a signal pulse of one polarity extends from time $t=0$ to time $t=T$ the total energy input is

$$W_1 = \int_0^T p_1 dt = E_B \int_0^T i_P dt.$$

Assuming for generality that a transformer is used having a transformation ratio n

$$W_1 = \frac{E_B}{n} \int_0^T i_L dt. \tag{3}$$

Let the load resistance be R_L and the maximum values of load voltage and load current be E_M and I_M, respectively. The total energy output is

$$W_2 = \frac{E_M}{I_M} \int_0^T i_L^2 dt \tag{4}$$

and the efficiency is

$$\frac{P_2}{P_1} = \frac{W_2}{W_1} = \frac{nE_M}{E_B I_M} \cdot \frac{\int_0^T i_L^2 dt}{\int_0^T i_L dt}. \tag{5}$$

Let I_{AV} and I_{RMS} be the average and root-mean-square values of the load current over the time interval 0 to T. Then

$$\frac{P_2}{P_1} = \left(\frac{nE_M}{E_B}\right)\left(\frac{I_{RMS}^2}{I_M I_{AV}}\right). \tag{6}$$

The first term nE_M/E_B depends upon how nearly the plate-voltage drop at maximum load voltage (the difference between E_B and nE_M) can be brought to zero. It will be convenient to have a name and symbol for this factor and it will therefore be called the voltage-utilization factor A. That is,

$$A = \frac{nE_M}{E_B}. \tag{7}$$

In terms of this factor, (3) becomes

$$W_1 = \frac{E_M}{A} \int_0^T i_L dt. \tag{8}$$

The second term depends only on the wave form. An alternative form for this function can be derived in terms of the ratio i_L/I_M. Writing u for this ratio, (8) and (4) become

$$W_1 = \frac{E_M I_M}{A} \int_0^T u dt \tag{9}$$

$$W_2 = E_M I_M \int_0^T u^2 dt \tag{10}$$

giving

$$\frac{P_2}{P_1} = \frac{W_2}{W_1} = A \frac{\int_0^T u^2 dt}{\int_0^T u dt}. \tag{11}$$

Since u is less than or equal to unity, u^2 is less than or equal to u. Therefore the function

$$\left(\frac{I_{RMS}^2}{I_M I_{AV}}\right) \equiv \frac{\int_0^T u^2 dt}{\int_0^T u dt} \leqq 1. \tag{12}$$

It can equal unity only for a rectangular wave pulse in which $i = I_M$ and $u = 1$ for the entire pulse.

Constant-Current Parallel Feed

The formula corresponding to (6) for efficiency in the case of constant-current parallel feed is

$$\frac{P_2}{P_1} = \left(\frac{n' I_M}{I_B}\right)\left(\frac{E_{RMS}^2}{E_M E_{AV}}\right). \tag{13}$$

The transformation ratio n' is the ratio of input current to load current, and the factor $n'I_M/I_B$ is the ratio of load current to supply current. If we call this factor the current-utilization factor A', it will be maximized by having the anode current (the difference between I_B and $n'I_M$) as small as possible at maximum load current.

For a purely resistive load the wave-form functions for the series and parallel feed cases are equal; that is,

$$\left(\frac{E_{RMS}^2}{E_M E_{AV}}\right) = \left(\frac{I_{RMS}^2}{I_M I_{AV}}\right). \tag{14}$$

Appendix II

Amplifier Efficiency with Time Subdivision of Wave

General Formulas

Assume that at times $T_1, T_2 \cdots T_K$, etc., the amplifier load is switched from branch 1 to 2, 2 to 3, $\cdots K$ to $K+1$, etc. Two cases will be considered; namely, load-impedance switching as illustrated in Figs. 4 and 9, and supply-voltage switching as illustrated in Fig. 11. It will be noted that the circuit using supply-voltage switching is not so convenient, as it requires the use of a power supply in which both terminals are above ground potential.

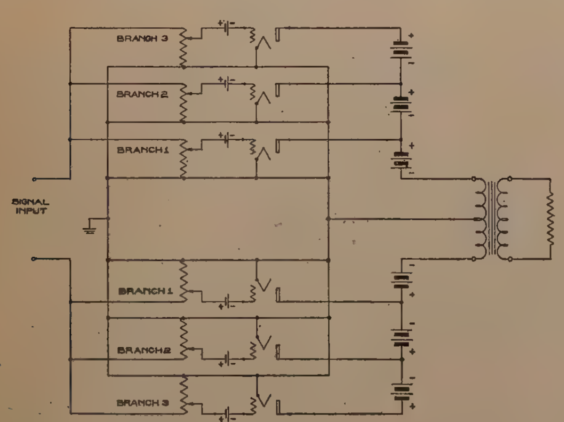

Fig. 11—Schematic circuit for switching supply voltage.

Let E_{MK} be the maximum load voltage during the time interval $t = T_{K-1}$ to $t = T_K$ that branch K is operating, and let

$$E_{MK}/E_M = V_K$$

in which E_M is the maximum load voltage for the entire wave. It should be noted that V_K is less than unity for all branches except the one operating on the peak voltage E_M.

By (7) of Appendix I, if neither load-impedance nor supply-voltage switching were used, the voltage-utilization factor A_K for the Kth branch would be

$$A_K = \frac{nE_{MK}}{E_B} = V_K A.$$

If it is assumed that the maximum voltage-utilization factor can be the same for all branches, then either the supply voltage can be decreased by the factor V_K, or the voltage-transformation factor increased by the reciprocal of this factor. That is, either

$$\frac{E_{BK}}{E_B} = V_K \tag{15}$$

or

$$\frac{n_K}{n} = \frac{1}{V_K}. \tag{16}$$

In either case, this will make the voltage-utilization factor for the Kth branch equal to A. That is

$$\frac{1}{V_K}\left(\frac{nE_{MK}}{E_B}\right) = \left(\frac{n_K E_{MK}}{E_B}\right) = \left(\frac{nE_{MK}}{E_{BK}}\right) = A. \tag{17}$$

By (8) the energy input to the Kth branch during the time interval $t = T_{K-1}$ to $t = T_K$ will be

$$W_{1K} = \frac{E_{MK}}{A_K}\int_{T_{K-1}}^{T_K} i_L dt. \tag{18}$$

If A_K is made equal to A either by reducing E_B to E_{BK} or by increasing n to n_K, the energy input becomes

$$W_{1K} = \frac{V_K E_M}{A}\int_{T_{K-1}}^{T_K} i_L dt. \tag{19}$$

Thus the energy input required during the time interval T_{K-1} to T_K has been reduced compared with that required with a conventional amplifier by the factor V_K.

Application to a Triangular Wave

Consider a triangular-shaped wave, that is, one in which the voltage and current increase linearly with time from zero to a maximum and then decrease linearly to zero again. During the phase of increasing current let

$$i_L = mt$$

and let i_L reach its maximum value at $t = T_M$ so that

$$I_M = mT_M.$$

Using (8) of Appendix I, the energy input without branch subdivision would be

$$W_1 = \frac{E_M}{A}\int_0^{T_M} mtdt$$

$$W_1 = \frac{E_M I_M T_M}{2A} \tag{20}$$

and the energy output either with or without branch subdivision will be, by (4)

$$W_2 = \frac{E_M}{I_M}\int_0^{T_M} m^2t^2dt$$

$$W_2 = \frac{E_M I_M T_M}{3}. \tag{21}$$

Therefore, without subdivision, the efficiency is

$$\frac{W_2}{W_1} = \frac{2}{3}A = 67\,A \text{ per cent.}$$

Assume now that the load is switched from branch 1 to branch 2 when $i = V_1 I_M$. The time at which i reaches this value is $T_1 = V_1 T_M$. Using (19), the input to branch 1 will be

$$W_{11} = \frac{V_1 E_M}{A}\int_0^{T_1} i_L dt$$

$$W_{11} = W_1 V_1^3. \tag{22}$$

Similarly, the energy input to branch 2 will be

$$W_{12} = \frac{E_M}{A}\int_{T_1}^{T_M} i_L dt$$

$$W_{12} = W_1(1 - V_1^2). \tag{23}$$

This gives for the total energy input

$$W_{11} + W_{12} = W_1(1 - V_1^2 + V_1^3). \tag{24}$$

By differentiating with respect to V_1 it will be found that for a minimum $V_1 = 2/3$. Using this value for V_1 gives for the energy input 0.85 W_1 and for the efficiency 78.3 A per cent.

Difference Equation for Determining Optimum Subdivision of Triangular Wave

The energy input for the Kth branch will be

$$W_{1K} = \frac{E_K}{A}\int_{T_{K-1}}^{T_K} idt$$

$$W_{1K} = \frac{E_M I_M T_M}{2A}(V_K^3 - V_{K-1}^2 V_K) \tag{25}$$

and for the $(K+1)$th branch

$$W_{1,K+1} = \frac{E_M I_M T_M}{2A}(V_{K+1}^3 - V_K^2 V_{K+1}). \tag{26}$$

The combined energy input to the Kth and $(K+1)$th branches is

$$W_{1K} + W_{1,K+1} = W_1(V_K^3 - V_{K-1}^2 V_K + V_{K+1}^3 - V_K^2 V_{K+1}).$$

If V_{K+1} and V_{K-1} are considered as fixed and V_K as variable, the value of V_K which will result in minimum energy input into the two branches is found by differentiating with respect to V_K and equating to zero. This gives

$$3V_K^2 - V_{K-1}^2 + 2V_K V_{K+1} = 0. \tag{27}$$

If V_K is assigned the value required to satisfy this equation, it can then be considered as a difference equation expressing the condition to be satisfied for minimum power input and maximum efficiency.

Transposing, we have

$$V_{K+1} = \frac{3}{2}V_K - \frac{1}{2}\frac{V_{K-1}^2}{V_K}.$$

A more convenient form for this difference equation can be obtained as follows: Let

$$\frac{V_{K+1}}{V_K} = Q_K.$$

Then

$$Q_{K+1} = \frac{3}{2} - \frac{1}{2Q_K^2}. \tag{28}$$

With two branches $Q_1 = 1/V_1$. Since V_1 has already been determined, Q_1 is known and values of successive Q's are readily calculated. The first five are

$$Q_1 = 1.500$$
$$Q_2 = 1.278$$
$$Q_3 = 1.194$$
$$Q_4 = 1.15$$
$$Q_5 = 1.12.$$

Power-Input and Output Formulas

A formula for the total energy input to N branches subdivided for optimum efficiency can be obtained as follows:

Let the energy input be

$$\sum_1^N W_{1K} = C_N \frac{E_N I_N T_N}{2A} \tag{29}$$

where the coefficient C_N is to be determined. For $N-1$ branches we have

$$\sum_1^{N-1} W_{1K} = C_{N-1}V_{N-1}^3\left(\frac{E_N I_N T_N}{2A}\right)$$

$$= \left(\frac{C_{N-1}}{Q_N^3}\right)\left(\frac{E_N I_N T_N}{2A}\right) \tag{30}$$

and for the Nth branch

$$W_{1N} = \frac{E_N I_N T_N}{2A}\left(1 - \frac{1}{Q_{N-1}^2}\right). \tag{31}$$

Equating (29) to the sum of (30) and (31) gives

$$C_N = \left(\frac{C_{N-1}}{Q_N^3} + 1 - \frac{1}{Q_{N-1}^3}\right).$$

Differentiating gives, for a minimum value of C_N,

$$Q_{N-1} = \frac{3}{2} C_{N-1}$$

so that

$$C_N = \frac{2}{3} \frac{1}{Q_N}. \qquad (32)$$

Substituting in (29) gives

$$\sum_{1}^{N} W_{1K} = \frac{2}{3} \frac{W_1}{Q_N}. \qquad (33)$$

From (21) and (29) the efficiency with optimum subdivision is

$$\frac{W_2}{W_1} = \frac{2}{3} \frac{A}{C_N} = \frac{A}{Q_N}. \qquad (34)$$

The power input to the Kth branch, from (31), is

$$W_{1K} = W_1 V_K^3 \left\{ 1 - \frac{1}{Q_{K-1}^2} \right\} \qquad (35)$$

and the power output, by integration of (4), is

$$W_{2K} = W_2 V_K^3 \left\{ 1 - \frac{1}{Q_{K-1}^3} \right\}. \qquad (36)$$

Theoretical Results for 1000-Watt Amplifier—Triangular Wave

Table I gives theoretical efficiencies, branch-voltage factors, and power distribution between branches, calculated from the foregoing formulas with from one to five branches. The voltage-utilization factor A was taken as 0.9, and the output power as 1000 watts in all cases.

TABLE I

One Branch—Efficiency 60 per cent						*Total*
Power Input—watts						1670
Power Output—watts						1000
Plate Dissipation—watts						670
Two Branches—Efficiency 70.5 per cent	*Branch 1*	*Branch 2*				*Total*
Voltage Factors (V_k)	0.67	1.00				
Power Input—watts	490	930				1420
Power Output—watts	300	700				1000
Plate Dissipation—watts	190	230				420
Three Branches—Efficiency 75.4 per cent	*Branch 1*	*Branch 2*	*Branch 3*			*Total*
Voltage Factors (V_k)	0.47	0.78	1.00			
Power Input—watts	240	440	650			1330
Power Output—watts	140	340	520			1000
Plate Dissipation—watts	100	100	130			330
Four Branches—Efficiency 78.5 per cent	*Branch 1*	*Branch 2*	*Branch 3*	*Branch 4*		*Total*
Voltage Factors (V_k)	0.44	0.66	0.84	1.00		
Power Input—watts	140	265	375	500		1280
Power Output—watts	85	200	300	415		1000
Plate Dissipation—watts	55	65	75	85		280
Five Branches—Efficiency 80.4 per cent	*Branch 1*	*Branch 2*	*Branch 3*	*Branch 4*	*Branch 5*	*Total*
Voltage Factors (V_k)	0.38	0.57	0.73	0.87	1.00	
Power Input—watts	90	170	260	320	410	1250
Power Output—watts	60	130	205	265	340	1000
Plate Dissipation—watts	30	40	55	55	70	250

Application to a Sinusoidal Wave

Let the load current during the time interval $t=0$ to $t=T_M$ be

$$i_L = I_M \sin \frac{\pi}{2}\left(\frac{t}{T_M}\right) = I_M \sin \theta. \qquad (37)$$

The input energy will be

$$W_1 = \frac{2}{\pi A} E_M I_M T_M \qquad (38)$$

and the output energy

$$W_2 = \tfrac{1}{2} E_M I_M T_M \qquad (39)$$

giving for the efficiency without subdivision

$$\frac{W_1}{W_2} = \frac{\pi A}{4}. \qquad (40)$$

Next assume subdivision between two branches with change-over occurring at $i=\sin(\pi/2\alpha)=\sin\theta_\alpha$ so that $T_1=\alpha T_M$. The input energy for branch 1 will be

$$W_{11} = W_1(1 - \cos\theta_\alpha)\sin\theta_\alpha$$

and for branch 2

$$W_{12} = W_1 \cos\theta_\alpha$$

giving for the total input

$$W_{11} + W_{12} = W_1(\sin\theta_\alpha - \sin\theta_\alpha\cos\theta_\alpha + \cos\theta_\alpha).$$

By differentiation it will be found that for maximum efficiency $\theta_\alpha=45$ degrees so that the voltage factor V_1 is 0.707.

Table II gives efficiencies and voltage factors for various values of θ_α. In each case the factor A was taken as 0.9.

TABLE II

EFFICIENCY OF TWO-BRANCH CIRCUIT WITH SINUSOIDAL WAVE

θ_α	Voltage Factor	Efficiency
Degrees		Per Cent
0	0	70.7
15	0.26	72.5
30	0.50	75.8
45	0.71	77.5
60	0.87	75.8
75	0.97	72.5
90	1.00	70.7

Difference Equation for Determining Optimum Subdivision of Sinusoidal Wave

Proceeding as in the case of the triangular wave, the energy input to the Kth branch is found to be

$$W_{1K} = \frac{2}{\pi A} E_M I_M T_M \left[\sin\theta_n(\cos\theta_{n-1} - \cos\theta_n)\right].$$

The energy input to the Kth and $(K+1)$th branches is

$$W_{1K} + W_{1,K+1} = \frac{2}{\pi A} E_M I_M T_M \left[\sin\theta_n(\cos\theta_{n-1} - \cos\theta_n) + \sin\theta_{n+1}(\cos\theta_n - \cos\theta_{n+1})\right].$$

Differentiating with respect to θ_n and equating to zero gives, for minimum energy input,

$$\cos\theta_n \cos\theta_{n-1} - \cos 2\theta_n - \sin\theta_n \sin\theta_{n+1} = 0 \quad (41)$$

from which

$$\sin\theta_{n+1} = 2\sin\theta_n - \frac{1 - \cos\theta_n \cos\theta_{n-1}}{\sin\theta_n}. \quad (42)$$

Power Output per Branch

The power output of the Kth branch will be

$$W_{2K} = \frac{E_M}{I_M}\int_{T_{K-1}}^{T_K} I_M{}^2 \sin^2\left(\frac{t}{T_M}\cdot\frac{\pi}{2}\right)dt$$

$$W_{2K} = \frac{E_M I_M T_M}{\pi}\left[(\theta_K - \theta_{K-1}) - \frac{\sin 2\theta_K - \sin 2\theta_{K-1}}{2}\right]. \quad (43)$$

Theoretical Results for 1000-Watt Amplifier—Sinusoidal Wave

Table III gives theoretical efficiencies, branch-voltage factors and power distribution between branches, calculated from the foregoing formulas with from one to five branches. The factor A was taken as 0.9 and the output power as 1000 watts throughout.

Effect on Power Input of Finite Time Interval for Branch Switching

The change-over from branch to branch will not in practice be instantaneous. In order to examine the effect that may be expected from a finite time interval for the transfer, let us assume that the power input into branch K falls at a uniform rate from $W = E_K I_A$ at $t = T_K - \Delta T$ to 0 at $t = T_K + \Delta T$, and that the power in branch $K+1$ increases uniformly from 0 at $t = T_K - \Delta T$ to $E_{K+1} I_B$ at $t = T_K + \Delta T$.

It follows that the power input to branch K during the transition interval is

$$p_K = E_K\left(\frac{I_A}{2} + \frac{t - T_K}{\Delta T}\cdot\frac{I_A}{2}\right).$$

The energy input during the interval will be

$$\int_{T_K - \Delta T}^{T_K + \Delta T} p_K dt = E_K I_A \Delta T.$$

Similarly, the energy input to branch $K+1$ during the same interval will be

$$\int_{T_K - \Delta T}^{T_K + \Delta T} p_{K+1} dt = E_{K+1} I_B \Delta T.$$

The total energy input will be

$$\Delta W_1 = (E_K I_A + E_{K+1} I_B)\Delta T.$$

With instantaneous change-over, the energy input during the same time interval, assuming that I changes linearly during the short time interval, would be

$$\Delta W_1' = \left[E_K\left(\frac{I_A + I_K}{2}\right) + E_{K+1}\left(\frac{I_K + I_B}{2}\right)\right]\Delta T.$$

Let $I_K - I_A = I_B - I_K = \Delta I$. Then the difference in energy input due to the finite time interval of change-over will be

$$\Delta W_1 - \Delta W_1' = \left(\frac{E_{K+1} - E_K}{2}\right)\Delta I \cdot \Delta T.$$

Thus a small increase in energy input or a drop in efficiency may be expected during the transition interval. Since the filter in the power-supply circuit tends to prevent fluctuations in power input, the drop in efficiency can be expected to be reflected mainly in a momentary drop in output during the switching interval. This is indicated in Fig. 6.

It will be noted that, although the actual efficiencies achieved in practice, as shown in Fig. 8, are less than the theoretical values derived in this Appendix, the rapid increase in efficiency obtained with the first two or three branches followed by a less marked improvement with additional branches is quite similar in the two cases.

A study of the voice-modulated wave is being made, and it is hoped to present supplementary data on the most desirable subdivision and the improvement in efficiency to be expected with voice waves in a later paper.

TABLE III

One Branch—Efficiency 70.6 per cent						*Total*
Power Input—watts						1412
Power Output—watts						1000
Plate Dissipation—watts						412
Two Branches—Efficiency 77.4 per cent	*Branch 1*	*Branch 2*				*Total*
Voltage Factor (V_k)	0.71	1.00				
Power Input—watts	293	1000				1293
Power Output—watts	182	818				1000
Plate Dissipation—watts	111	182				293
Three Branches—Efficiency 80.3 per cent	*Branch 1*	*Branch 2*	*Branch 3*			*Total*
Voltage Factor (V_k)	0.57	0.83	1.00			
Power Input—watts	138	319	790			1247
Power Output—watts	90	240	670			1000
Plate Dissipation—watts	48	79	120			247
Four Branches—Efficiency 82.4 per cent	*Branch 1*	*Branch 2*	*Branch 3*	*Branch 4*		*Total*
Voltage Factor (V_k)	0.485	0.71	0.88	1.00		
Power Input—watts	81	174	289	666		1210
Power Output—watts	52	134	237	577		1000
Plate Dissipation—watts	29	40	52	89		210
Five Branches—Efficiency 83.8 per cent	*Branch 1*	*Branch 2*	*Branch 3*	*Branch 4*	*Branch 5*	*Total*
Voltage Factor (V_k)	0.43	0.63	0.785	0.91	1.00	
Power Input—watts	57	114	174	263	586	1194
Power Output—watts	34	85	144	223	514	1000
Plate Dissipation—watts	23	29	30	40	72	194

The Transverse Electric Modes in Coaxial Cavities*

ROBERT A. KIRKMAN† AND MORRIS KLINE†

Summary—Some thought on the transverse electric modes in resonant coaxial cavities labeled $TE_{1,0,1}$, $TE_{1,0,2}$, $TE_{2,0,1}$, $TE_{3,0,1}$, etc., by Barrow and Mieher[1] suggested several conclusions which are perhaps implicit in their paper but which deserve explicit consideration. In addition, the notation and the diagrams of the electric field configurations of these modes, as presented in that reference, cause misconceptions and confusion which subsequent papers and even textbooks are perpetuating.[2] Actually, the transverse electric modes whose middle subscript is zero do not exist. They are limiting cases and are approached by the fields of the coaxial modes $TE_{1,1,1}$, $TE_{1,1,2}$, $TE_{2,1,1}$, $TE_{3,1,1}$, etc., respectively, as the ratio of the radii of the inner and outer conductors approaches 1. Several facts about the behavior of these modes for varying values of this ratio are presented. In particular, for a given mode, the resonant frequency of a coaxial cavity decreases as the ratio increases. In the case of a cavity of infinite length (i.e., a wave guide) the corresponding wavelengths (i.e., the critical wavelengths of the guides) approach the circumference of the cavity divided by the first subscript of the mode. Physical and mathematical arguments confirm these conclusions and make clear to what extent the Barrow and Mieher diagrams of the modes $TE_{1,0,1}$, $TE_{1,0,2}$, etc., are representative of actual coaxial modes. The practical importance of the transverse electric coaxial modes in ultra-high-frequency work is emphasized.

Introduction

THE TRANSVERSE electric modes of coaxial cavities are usually designated by the notation $TE_{1,1,1}$, $TE_{2,1,1}$, etc., and, in general, by $TE_{l,m,n}$ or $H_{l,m,n}$ wherein the subscripts l, m, and n have both mathematical and physical significance. The mathematical significance centers in the fact that the calculation of the resonant frequencies of a coaxial cavity of given dimensions uses the roots of the equations

$$J_l'(x)Y_l'(\rho x) - J_l'(\rho x)Y_l'(x) = 0 \quad (l = 0, 1, 2, 3, \cdots) \quad (1)$$

where J_l' and Y_l' $(=N_l')$ are the derivatives with respect to their arguments of the lth order Bessel functions of the first and second kind, respectively, and ρ is the ratio of the radii a and b of the inner and outer conductors of the cavity. Hence, the first subscript l in the notation $TE_{l,m,n}$ states the order of the Bessel functions of the first and second kind which must be used to calculate the resonant frequency of that mode; that is, it selects the l value in (1). The second subscript m in the designation of the mode denotes the order of magnitude of that root of (1) which is used to calculate the resonant frequency. The third subscript n is merely a coefficient in the argument of a trigonometric function which enters into the expressions for the electric and magnetic fields inside the cavity.

The physical significance of these subscripts is usually purported to be as follows: At any point in a cavity (and at a given instant) the electric field has a definite direction and magnitude. This electric field vector can be resolved into three independent components. In the case of a coaxial cavity, the directions of these components are chosen to be (1) along the circle through the point and concentric with the inner and outer conductors, (2) along the radius through the point, (3) along a line running through the point and lengthwise along the cavity. These components are usually designated as E_ϕ, E_r, and E_x because cylindrical co-ordinates are employed to study the field mathematically. The subscript l of $TE_{l,m,n}$ is supposed to denote physically the number of full periods of sinusoidal variation in E_r along any circle concentric with the inner and outer conductors. The subscript m is supposed to denote the number of half periods of sinusoidal variation in E_ϕ along a radius. The subscript n denotes the number of half periods of sinusoidal variation in E_r along the length of the cavity.

In view of the mathematical meanings of the subscripts l, m, n, the notation $TE_{1,0,1}$, $TE_{2,0,1}$, etc., is mystifying because the calculation of the resonant frequencies of cavities of given dimensions supporting such modes would call for the zeroth roots of (1) whose roots in order of magnitude are usually designated as first, second, third, etc. Moreover, while the pictures of the electric fields of these modes as presented by Barrow and Mieher are consistent with the physical meanings usually offered for the subscripts l, m, and n the pictures are strictly inconsistent with facts of electromagnetic theory. Finally, the physical meanings usually assigned to the subscripts l, m, and n do not appear to be applicable to all coaxial electric field configurations.

For these reasons it was decided to survey the transverse electric modes in coaxial cavities and, in particular, to study the variation in the electric fields of these modes as the ratio of the radii of the inner and outer conductors is varied. The results of this survey do clear up the above difficulties.

The Variation in the Electric Fields of the $TE_{l,m,n}$ Modes with the Ratio of the Radii

The mathematical functions which represent the three electric components and the three magnetic components of the electromagnetic field of the various modes inside a resonant coaxial cavity involve the roots[3] of (1). For example, the expressions for the field in a coaxial cavity supporting the $TE_{2,1,1}$ mode involve the first root of that member of (1) corresponding to $l=2$. Since both $J_l'(x)$ and $Y_l'(x)$ are infinite series, these roots are not readily obtainable even for a definite value of ρ, the ratio of the radii. Yet, the variation in these roots with ρ is precisely what is required in order to study the variation in the modes with changing ratio of the radii. Graphing must be utilized.

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[1] W. L. Barrow and W. W. Mieher, "Natural oscillations of electrical cavity resonators," Proc. I.R.E., vol. 28, pp. 184–191; April, 1940.

[2] R. I. Sarbacher and W. A. Edson, "Hyper and UHF Engineering," John Wiley and Sons, Inc., New York, N. Y., 1943, p. 390.

[3] F. Borgnis, "Die Konzentrische Leitung als Resonator," *Hochfrequenz. und Elektroakustik*, vol. 56, pp. 47–54; August, 1940.

The graphical procedure has been considerably simplified by Truell[4] and Fig. 2 of his paper shows, with appropriate changes in notation, the variation of $x_{l,1}$; that is, the first root of (1), with ρ the ratio of the radii a and b of the inner and outer conductors.

To study the variation in the transverse electric modes with varying a/b let us select the $TE_{3,1,1}$ mode as an illustration. When a/b is 0, we have the pure cylindrical cavity rather than the coaxial one. The electric fields of $TE_{l,m,n}$ modes in cylindrical cavities are known and are given by Barrow and Mieher, among others. The field of $TE_{3,1,1}$ is reproduced in Fig. 1 for comparison with later figures.

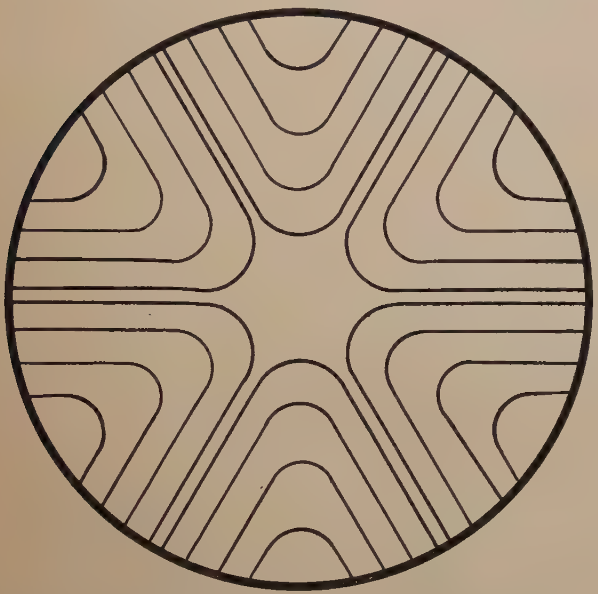

Fig. 1—Electric field for $TE_{3,1,1}$ mode in a perfect cylinder.

When a/b is small, the field inside the coaxial cavity is much like the field inside the cylinder, except that a few of the electric lines terminate on the inner conductor. In fact, the first root of (1) for small ρ, i.e., small a/b, is practically the first root of $J_l'(x)=0$, ($l=0, 1, 2, \cdots$), the equations whose roots play the same part for cylindrical cavities that the roots of (1) do for coaxial cavities. Hence it follows from the mathematical expressions in Borgnis[5] that the field in the coaxial cavity is, for small a/b, nearly the same as in a cylindrical cavity. The larger the center conductor the fewer electric lines loop back without touching it and the more go directly to the inner conductor (see Fig. 4).

Let us pay close attention to the case where the radius of the inner conductor approaches the radius of the outer one (Fig. 2). If a metal sheet is placed inside a cavity so that it is perpendicular at any point to the electric line through that point, the field will not be disturbed. Hence, we may cut the cavity of Fig. 2 by means of imaginary sheets *cd*, *ef*, *gh*, etc., into a number of nearly rectangular cavities without disturbing the field. Because the component of the electric field which parallels the inner and outer conductors is zero or small almost everywhere in the cavity, we have in each "rectangle" approximately the field which exists in a true rectangular cavity carrying the $TE_{1,0,1}$ mode.[6] In the true rectangular cavity, if the electric lines are vertical, the vertical dimension is arbitrary though the horizontal dimensions are not. Hence, in the coaxial cavity with the inner radius a nearly equal to the outer radius b independence of the dimension $b-a$ is practically attained. However, since the coaxial cavity consists of "rectangles" side by side, the circumference of the cavity is still decisive in determining the resonant frequency.

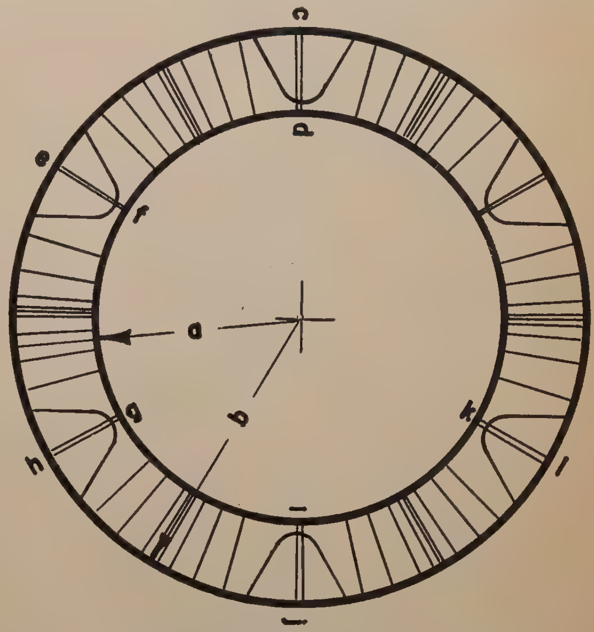

Fig. 2—Electric field for coaxial $TE_{3,1,1}$ mode with a nearly equal to b.

Conclusions on the Limiting Wavelength as a/b Approaches 1

The analysis of the electric field in a coaxial cavity for which a/b approaches 1 in terms of rectangles leads directly to conclusions about the limiting wavelength. Before stating them, let us shift the burden of the discussion from cavities to wave guides. With increasing length, the transverse dimensions of a resonant cavity approach the critical or cutoff dimensions of a guide with the same cross-sectional shape carrying the same mode. Hence a discussion of guides is, in effect, a discussion of cavities.

In a guide of rectangular cross section carrying the $TE_{1,0}$ mode, one dimension is arbitrary and the critical size for the other is one half the free-space wavelength. As Fig. 2 shows, increasing a/b in the coaxial guide tends to produce, in each half-period variation of the radial component of the electric field, the same field as exists in the $TE_{1,0}$ mode of a rectangular guide. (There are six half-period variations in Fig. 2, the number being always $2l$.) The critical value of the circumference for the $TE_{l,1}$ modes of a coaxial guide with a/b nearly 1

[4] R. Truell, "Concerning the roots of $J_n'(x)N_n'(kx)-J_n'(kx)N_n'(x)=0$," *Jour. Appl. Phys.*, vol. 14, pp. 350–352; July, 1943.

[5] See equations (2) and (6), pp. 48–49 of footnote reference 3.

[6] See, for example, Fig. 6.3, of footnote reference 2.

should therefore be $2l$ free-space half wavelengths or, for a fixed circumference, the critical wavelength in the $TE_{l,1}$ modes approaches the circumference divided by l.

Calculation supports the conclusion drawn in the preceding paragraph. The critical wavelength for a coaxial guide carrying the $TE_{l,1}$ mode[7] is

$$\lambda_0 = \frac{c}{f_0} = \frac{2\pi b}{x_{l,1}}. \tag{2}$$

Fig. 2 of Truell's paper shows that $x_{3,1}$, for example, approaches 3 as a/b approaches 1. Hence the critical wavelength approaches 1/3 of the circumference.[8]

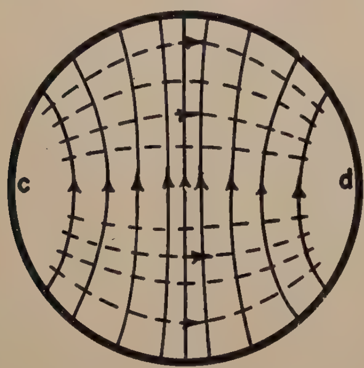

Fig. 3—Electric (solid) and magnetic (broken) lines for $TE_{1,1}$ mode of a circular guide.

As Truell's Fig. 2 illustrates, $x_{l,1}$ decreases as a/b approaches 1. (Note that his k is the reciprocal of our ρ.) It follows from (2), then, that with a fixed outer circumference by increasing a/b we increase the critical wavelength; i.e., the guide is able to pass more frequencies. Thus decreasing the size of the guide in this manner has an effect opposite to decreasing the outer circumference or to decreasing the length in a cavity.

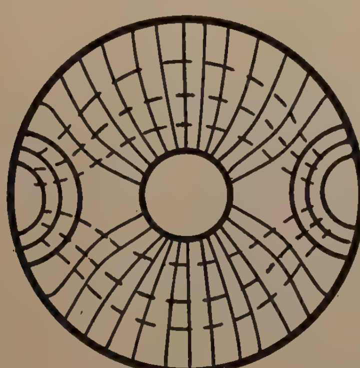

Fig. 4—Electric (solid) and magnetic (broken) lines for $TE_{1,1}$ mode of a coaxial guide.

It is instructive to view and confirm physically what happens when the inner conductor increases in size. The closed magnetic loops which are present in all waveguide and cavity modes are incompressible below a minimum "dimension" of a half wavelength. They do not permit themselves to be "squeezed" into narrower regions. Consider, for example, the $TE_{1,1}$ mode. When a/b is 0 the guide is cylindrical and the field is as shown in Fig. 3. The magnetic lines are directed out toward the reader at c, flow across the front, and head into the guide again at d. Hence the diameter of the guide cannot be less than one-half wavelength. As a matter of fact, the minimum diameter for this mode is 0.586 λ. The introduction of an inner conductor into the guide forces the magnetic loops to flow around it as shown in Fig. 4. Moreover, the loops curve more nearly into the shape of the outer circumference as the inner conductor is increased in size. This curvature permits a longer wavelength. The limiting case as the inner conductor increases calls for two loops, one above and one below the inner conductor, each loop about one-half wavelength

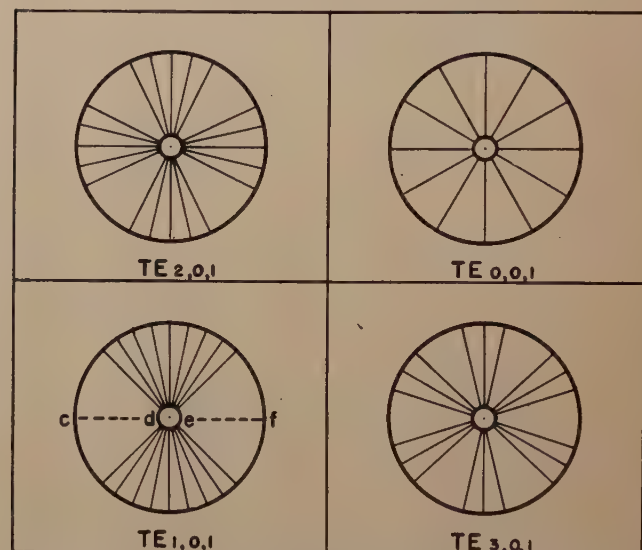

Fig. 5—Electric field lines of some transverse electric coaxial modes as portrayed by Barrow and Mieher.

in size, and each running along one portion of the circumference. Hence, the latter should be two half wavelengths long, which is the conclusion reached in the preceding paragraph.

It need not be remarked, perhaps, that the above discussion of the $TE_{3,1,1}$ coaxial cavity mode and the corresponding $TE_{3,1}$ guide mode applies to all the $TE_{l,1,n}$ coaxial cavity and corresponding guide modes.

The $TE_{l,0,n}$ Modes of Barrow and Mieher

It is now clear as to where the modes labeled $TE_{1,0,1}$, $TE_{2,0,1}$, and $TE_{3,0,1}$ by Barrow and Mieher and reproduced in Fig. 5 fit into the scheme of things. They are approximate representations of the $TE_{1,1,1}$, $TE_{2,1,1}$, and $TE_{3,1,1}$ modes, respectively, and are attained *practically only* for large a/b. The subscript 0, though descriptive of the field in that there is practically no variation in the component of the electric field paralleling a circle concentric with the inner and outer conductors as one proceeds radially, is nevertheless misleading.

[7] See equation (3) of footnote reference 1.

[8] One can assume at once that a coaxial wave guide in which the two radii are almost equal is practically a rectangular guide, and apply formulas for rectangular guides. This is done by S. A. Schelkunoff, "Electromagnetic Waves," D. Van Nostrand Co., Inc., New York, N. Y., 1943, p. 391. However, one should not be misled by the procedure adopted there into believing that there exist additional modes which are obtained by letting $m=0$; i.e., that $TE_{l,0}$ modes exist which are distinct from the $TE_{l,1}$ modes. (See the next section of this paper.)

There are mathematical and physical arguments which show that the fields in Fig. 5 are not, and undoubtedly were not intended to be, taken as exact descriptions. In cylindrical co-ordinates, the electric field inside a cavity can be resolved into the three components E_ϕ, E_r, and E_x. For the transverse electric modes, E_x is zero everywhere in the cavity. According to Fig. 5, the component E_ϕ which parallels circles concentric with the inner and outer conductors must also be zero. Now solution of Maxwell's equations for the field inside the cavity, under the conditions that E_x and E_ϕ are identically zero everywhere inside, shows that the resonant frequency should not change with the mode. Yet the resonant frequencies which Barrow and Mieher give for these modes, in the particular cavity they used, are 344 megacycles for the (1, 0, 1) mode, 475 megacycles for the (2, 0, 1) mode, and 611 megacycles for the (3, 0, 1) mode. Indeed a check on their calculations shows that Barrow and Mieher obtained these frequencies by treating the $TE_{l,0,n}$ modes as $TE_{l,1,n}$ modes and by using equation (1).

A physical argument for the contention that the diagrams in Fig. 5 are not exact is illuminating. Consider the $TE_{1,0,1}$ mode. According to the illustration, it should be possible to insert conducting sheets along the cavity passing through *cd* or *ef* or both, without disturbing the field. But if sheets were present, lines of force approaching them closely would be compelled by the usual boundary condition to curve and terminate upon the sheets (Figs. 2 and 4); or we might resort to a convention of electric field theory and say that electric lines act like stretched elastic bands; hence some will shorten themselves by curving and meeting lines symmetric with respect to *cd* and *ef*.

The Practical Importance of the Coaxial Transverse Electric Modes

It is urged that the coaxial transverse electric modes be clearly understood because of their relation to the principal or $TE_{0,0,1}$ mode (Fig. 5) which is commonly used in resonant coaxial lines, and which is but one special case of the infinite number of modes which can be sustained in a resonant coaxial cavity. If the frequency is low, the dimensions of the usual resonant coaxial line are such that other modes are not sustained. But as the frequency increases, particularly if the diameter of the inner conductor becomes comparable to that of the outer one, the resonant line may readily sustain the higher transverse electric modes; i.e., the $TE_{l,1,n}$ modes. The similarity of the fields (one has but to compare the fields in Fig. 5) makes it likely that the device designed to propagate the principal mode will also propagate the higher modes when they can be sustained.

As an example, a coaxial line whose outer radius is 2 centimeters will, of course, resonate in the principal mode of a 3000-megacycle wave. But it may also resonate in the $TE_{1,1,1}$ mode if the inner radius is greater than 0.6 of the outer radius. The higher the frequency the less the inner radius need be, or the greater the possibility of still higher modes being sustained. In view of the fact that frequencies much higher than 3000 megacycles are now well within the range of experimental work, the likelihood of a coaxial line sustaining or transmitting several modes where only one is intended is by no means negligible.

The introduction of these higher modes can, of course, produce intolerable effects. Even in a simple device like

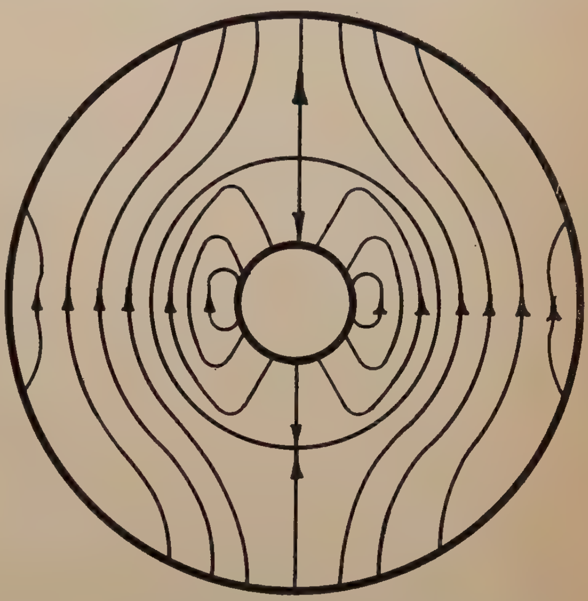

Fig. 6—Higher transverse electric coaxial mode $TE_{1,2,1}$, when a/b is between 0.2 and 1.

a wavemeter employing a resonant coaxial line, the peaks or dips of the principal mode may no longer be recognizable if higher modes are present or appear in the line as its length is altered by the usual tuning means.

It may be remarked finally that the l, m, n notation used in describing the modes of cavities has its limitations. Whereas the mathematical meanings still hold precisely, the physical meanings suggested by Barrow and Mieher[9] cannot be applied too literally. Reference to Figs. 1, 3, and 4 will show that the physical meaning assigned to the second subscript m does not hold for all radial paths. Moreover, various physical pictures must be associated with the same set of subscripts. As an example, the field shown in Fig. 6, and which is given by Borgnis[10] is the one we must associate with $TE_{1,2,1}$ as long as the ratio of a to b is at least 0.2. For ratios less than 0.2 the field has more variations in it. Even in the case pictured it is difficult to decide what the m value should be.

[9] See p. 185 of footnote reference 1.
[10] See p. 53 of footnote reference 3.

Radio-Frequency Spectrum Analyzers*

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Summary—**The resolving power of radio-frequency spectrum analyzers of the continuously tuned type is defined as the width in frequency, at points 3 decibels down, of the trace of a continuous-wave signal. The optimum resolving power is $1.3\sqrt{F/T}$, in which F is the frequency band scanned, and T the period of one scan. Traces of pulse-modulated, frequency-modulated, and amplitude-modulated signals are illustrated to show effect of resolving power.**

Introduction

IN THE absence of a standard definition, the term "radio-frequency spectrum analyzer" is considered to apply to a device which provides a description of signal distribution and sideband structure in a selected radio-frequency band in the form of a plot of amplitude versus frequency. In the parallel field of optics, spectrum analysis by means of spectroscopes is conducted for "the investigation of substances or bodies by means of their spectra";[1] in radio, it is assumed that spectrum analysis is applied to the investigation of radio signals by means of the traces observed on the radio-frequency spectrum analyzer.

In an optical spectroscope, all frequencies in a selected band are received simultaneously and split into groups. In an analogous manner a radio-frequency band can be divided into groups and analyzed by a series of fixed-tuned receivers staggered in frequency throughout the band. Although such devices provide more readily interpreted and reliable indications than the type to be described, their use is infrequent because of the multiplicity of circuits required.

Radio-frequency spectrum analysis may also be accomplished by the continuous tuning of a selective receiver through the spectrum band under study, examining each frequency group in turn for the existence of signals, rather than all groups in the band simultaneously. Such devices have been manufactured for some years and when used with synchronized cathode-ray-tube presentations are described commercially as "panoramic"[2] receivers. This paper is concerned only with the latter type of spectrum analyzer.

Fig. 1 shows the block diagram of a typical continuously tuned spectrum analyzer, a superheterodyne receiver periodically tuned over a band together with a synchronized display device. The oscillator control causes the oscillator frequency to vary approximately linearly in time so that the receiver tunes linearly over

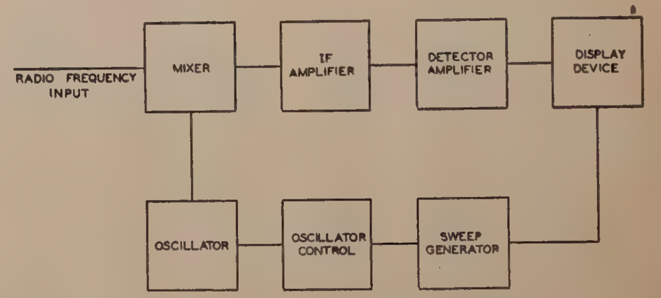

Fig. 1—Continuous-tuning spectrum analyzer.

the spectrum under examination. At the completion of a tuning sweep, the oscillator makes a rapid return to the original starting frequency during an interval in which the display device is blanked. The tuning cycle is then repeated. In many cases, the mixer is also tuned to inhibit spurious responses. In operation, the mixer output presents to the intermediate-frequency amplifier all continuous signals within the range of the analyzer with a superposed linear wide-range frequency variation added to any modulation originally in the signals. Separation of the band into individual signals occurs as the linear frequency variation causes these signals, in sequence, to tune through the intermediate-frequency-amplifier passband, and the degree of signal separation is determined entirely by the intermediate-frequency-amplifier properties. It is the purpose of this paper to discuss the "resolving," or signal separating powers, of continuously tuned spectrum analyzers, and the paper is concerned entirely with the phenomena encountered in the intermediate-frequency amplifier.

Because of the periodic sweep the intermediate-frequency-amplifier response in a spectrum analyzer can be expressed as the sum of the responses to a series representing the expansion into components of the cyclically frequency-modulated signal. The response can also be treated as a transient one, in which each transient rises and decays entirely during a single sweep period, because the case in which the transient persists from one complete tuning cycle to the next is one in which a (resolutionless) continuous display is produced, and is therefore trivial.

Neither method of analysis yields useful results for a generalized system. A particular intermediate-frequency-amplifier response for specific sweep widths can

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† Carnegie Institute of Technology, Pittsburgh, Pa.
[1] Webster's Collegiate Dictionary, Fifth Edition.
[2] Product of Panoramic Radio Corporation, New York, New York.

be computed (albeit tediously) and some form of generalized curves may eventually be available. However, experimental studies of the relation between resolving power and intermediate-frequency-amplifier bandwidth result in surprisingly simple empirical relations which have been widely applied in designs. These involve the following factors, for which convenient definitions have been chosen:

Resolving Powers (S)*:* This is the displayed width, in terms of frequency, at the 3-decibel-down points, of a continuous-wave signal. Thus an analyzer which presents a continuous-wave signal as a "pip" 10 kilocycles wide at the 3-decibel-down points would be said to have a resolving power of 10 kilocycles. Fig. 2 shows a typical trace used in determining resolving power. Two equal unmodulated signals differing in frequency by S defined in this manner would be barely separable.

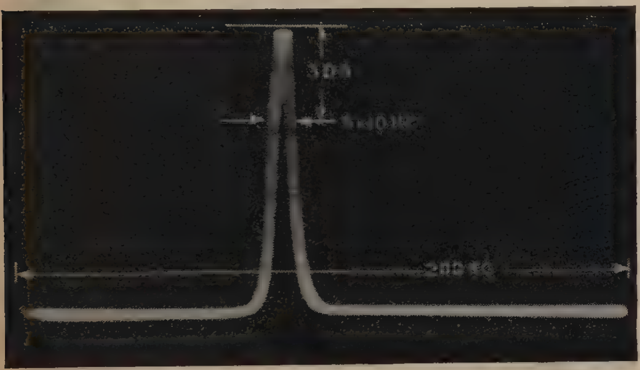

Fig. 2—Construction for determining resolution.

Intermediate-Frequency-Amplifier Bandwidth (Δf)*:* For consistency this is defined as the width, in cycles per second, of the intermediate-frequency selectivity curve between 3-decibel-down points.

Sweep Bandwidth of the Spectrum Analyzer (F)*:* This is the difference of maximum and minimum frequencies between which the analyzer tunes and it is assumed that this band is traversed linearly in time.

Sweep-Time Interval (T)*:* This is the time interval of one displayed sweep from minimum to maximum frequency (or vice versa if the direction of displayed frequency is from maximum to minimum) and does not include the return-trace time.

Experimentally it is found that maximum resolving power for a spectrum analyzer is very nearly

$$S = 1.3\sqrt{\frac{F}{T}} \tag{1}$$

and that this resolving power is realized for intermediate-frequency-amplifier bandwidths Δf in the vicinity of

$$\Delta f = \sqrt{\frac{F}{2T}}. \tag{2}$$

A somewhat intuitive analysis for a single resistance-inductance-capacitance circuit provides a substantiation for the dependence of bandwidth and resolving power on $\sqrt{F/T}$ alone.

If a tuned circuit is excited by a linearly frequency-modulated signal (of very much greater deviation that the bandwidth of the circuit) the resulting impulse consists of two components: (a) that at the natural circuit frequency; and (b) that at the instantaneous applied signal frequency.

If the circuit Q is very low, the transient (a) decays so rapidly that the output is a faithful trace of the normal intermediate-frequency-amplifier curve. If the circuit Q is very high, the response (a) is important. Its time is dependent on the circuit time constant and successive increases in Q beyond a certain point increase the time constant so much as to decrease resolution. Therefore it appears reasonable that the choice of a circuit bandwidth in which the time of steady-state response (b) is equal to the transient (a) rise and decay time (both times for 3-decibel-down points) should result in maximum resolving power.

For instance, let a frequency band F be swept in a time interval T. The bandwidth Δf (in cycles per second) of a single circuit tuned to frequency f_c is, from standard selectivity curve,

$$\Delta f \doteq \frac{f_c}{Q} \tag{3}$$

between 3-decibel points.

The circuit response at the instantaneous applied frequency will then occur during a time

$$\Delta t_1 = \frac{\Delta f}{F} \times T = \frac{f_c T}{QF}. \tag{4}$$

The circuit transient time constant for a variable-frequency applied signal is unknown; if, however, it is assumed approximately that for a fixed-frequency signal, the 3-decibel decay time Δt_2, is determined from

$$\frac{R}{2L}\Delta t_2 = -\log_e 0.707 \quad \text{or} \quad \Delta t_2 \approx \frac{Q}{2\omega}.$$

For a rise and decay of 3 decibels, the time is $\Delta t_3 = Q/\omega$.

If Δt_1 is equated to Δt_3 the bandwidth of the circuit is found to be

$$\Delta f = 0.56\sqrt{\frac{F}{2T}}. \tag{5}$$

The similarity of (2) and (5) needs no further comment.

Effect of Resolution on Sideband Traces

As would be anticipated, it is found that the ability of a spectrum analyzer to show sideband structure is measured by the resolving power and sidebands of equal

amplitude differing in frequency by S can be distinguished on an analyzer trace. If amplitude of adjacent sidebands is not equal, somewhat greater resolving power is required for their separation because of the masking of the smaller signal by the larger adjacent signal.

Modulated signal traces for resolution lower than that necessary for sideband separation are of considerable interest, particularly in the case of pulsed signals, as shown by the following analysis for a simplified circuit.

Let a series resistance-inductance-capacitance circuit tuned to a natural angular frequency ω_r be excited by a carrier of fixed angular frequency ω_a pulsed on and off at angular frequency ω_m with a pulse of duration d. The response to the first pulse is of the form

$$i = K_1 \epsilon^{-Rt/2L} \cos (\omega_r t + \psi) + K_2 \sin (\omega_a t + \phi) \qquad (6)$$

(K_1, K_2, ψ, ϕ determined by circuit constants and ω_a).

In the interval between the end of this first pulse and the incidence of the second there will remain only transient terms of frequency ω_r, and up to this time there is no dependence of transient amplitude upon pulse-repetition rate because the circuit has been excited by but a single pulse. If the transient from the first pulse is effectively decayed at the incidence of the second, the response to the second pulse will be exactly equal to that of the first. If, on the other hand, only a slight decay takes place between pulses, successive pulses will result in the modification of terms such as K_1 in (6) by amplitude functions of $\omega_a - \omega_r/\omega_m$ in which the amplitude is a maximum for $\omega_a - \omega_r/\omega_m = n$, an integer, corresponding to the usual relation, in which the sideband separation is the modulation frequency. Thus, in general, a tuned circuit is not capable of distinguishing (resolving) pulse-modulated-signal sidebands unless the transient resulting from each pulse persists at least until the next pulse appears. Fig. 3 illustrates the trace of a pulsed signal in which sidebands are resolved.

When the transient decay between pulses is large no sidebands are resolved; there may, however, be a well-defined pulse envelope. Consider the resistance-inductance-capacitance circuit for which (6) was developed; if the resistance is zero this reduces to

$$i = \frac{E}{z}\left(-\frac{\omega_r}{\omega_a}\sin\phi\sin\omega_r t + \cos\phi\cos\omega_r t - \cos\left[\omega_a t + \psi\right]\right) \qquad (7a)$$

and for values of R very near zero, it is approximately,

$$i \approx \frac{E}{z}\left[\epsilon^{-Rt/2L}\left(-\frac{\omega_r}{\omega_a}\sin\phi\sin\omega_r t + \cos\phi\cos\omega_r t\right) - \cos(\omega_a t + \psi)\right]. \qquad (7b)$$

At the end of time d, the pulse ceases. There will be no transient after this time if at the instant of pulse termination, the current i_d and the charge

$$q_d = \int_0^d i dt$$

are zero. For (7a) this is the case for

$$\omega_r \approx \omega_a \quad \text{and} \quad \omega_r d - \omega_a d = 2n\pi \qquad (8)$$

or

$$f_r - f_a = \frac{n}{d},$$

where n is an integer.

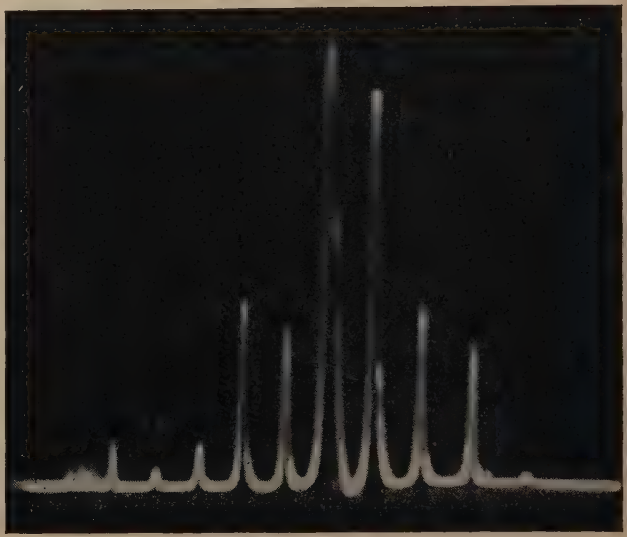

Fig. 3—Pulse-modulated signal of 75,000 pulses per second; $F=1$ megacycle; $S=10$ kilocycles; $T=1/40$ second; pulse duration $=3$ microseconds.

Although with (7b) there can be no simultaneous zeros of current and charge, a minimum i_d and q_d are reached for approximately the same condition (8) which is the usual relation, locating the nulls for the sideband envelope, following the form of the function $\sin x/x$. Appearance of the sideband envelope on the trace is therefore dependent only on the transient-response persistence for each individual pulse, and the sideband envelope cannot be distinguished on the analyzer trace if the term $\epsilon^{-Rt/2L}$ is very much less than 1. With short pulses the duration of a single pulse may be a small fraction of the interval between pulses and a rate of decay in an intermediate-frequency amplifier which is excessive for sideband resolution may easily be sufficiently low to permit sideband-envelope resolution. An estimate of the resolving power required to show this sideband envelope may be obtained by assuming an allowable decay and calculating the corresponding resolving power for the simple resistance-inductance-capacitance circuit.

If the allowable decay is assumed to be 10 decibels, the pulse duration d should be

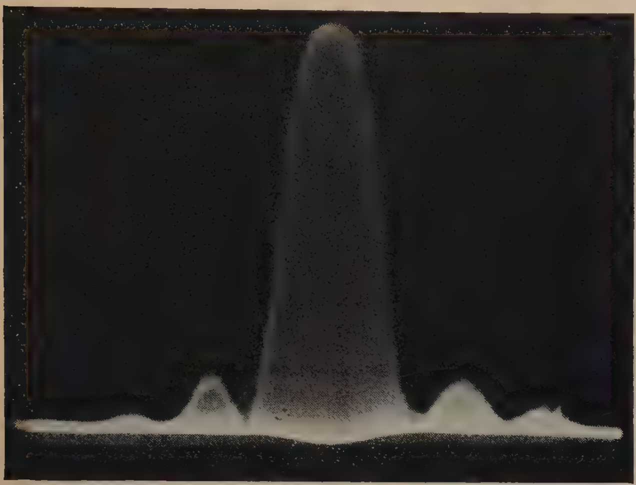

Fig. 4—Pulses of 14 microseconds; $F=300$ kilocycles; $S=10$ kilocycles; $T=1/40$ second.
(Top): 5000 pulses per second.
(Botton): 1600 pulses per second.

$$d < \frac{2L}{R} \log_e \frac{1}{10}$$

or

$$d < \frac{4.6L}{R} \cdot$$

If

$$Q = \frac{f}{\Delta f} \quad \text{and} \quad \Delta f = \sqrt{\frac{F}{2T}}$$

the duration d must be

$$d < 1.02\sqrt{\frac{T}{F}}$$

or the frequency separation F_s of nulls in the sideband envelope is

$$F_s > 1.96\sqrt{\frac{F}{T}} \tag{9}$$

for sideband-envelope resolution. Equation (9) is so

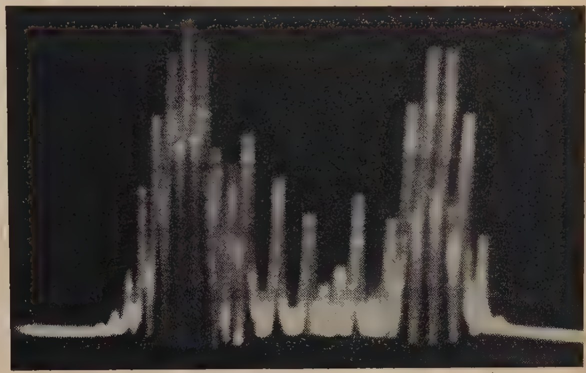

Fig. 5—Frequency-modulation signal, 200-kilocycle deviation; 20-kilocycle modulation; $S=10$ kilocycles; $F\approx 700$ kilocycles; $T=1/40$ second.

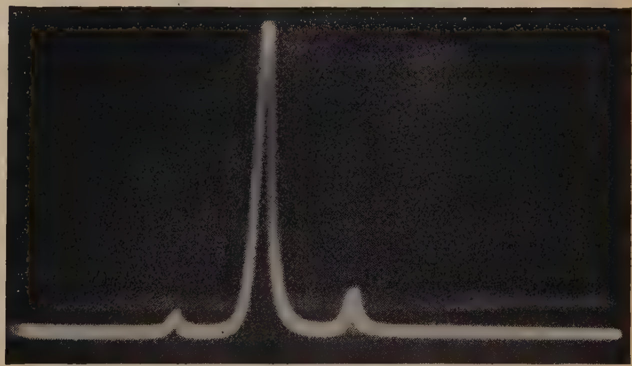

Fig. 6—Amplitude-modulation signal, 50-kilocycle modulation; $F=300$ kilocycles; $S=10$ kilocycles; $T=1/40$ second.

nearly the same as (1) that the difference in coefficient may be attributed to inaccuracies in assumptions and this conclusion is substantiated by experimental results. Fig. 4 illustrates two cases in which the sideband envelope is very well defined although individual sidebands are not resolved. In these illustrations the pulse rate was synchronized at a sweep-rate harmonic to facilitate photography. The vertical "spikes" should not be confused with sidebands, for which the horizontal separation is far too great. Each "spike" is a pulse and the time interval between pulses is equal to the sweep-time interval on the screen.

When resolution is very coarse, pulse-signal traces are decidedly ambiguous, except in the extreme cases (very coarse resolution) in which pulse signals appear as a single trace, "bobbing" up and down.

The traces of frequency-modulated (or phase-modu-

lated) and amplitude-modulated signals are not amenable to as straight-forward an explanation as those of pulse-modulated signals. In addition, neither angular-modulated- nor amplitude-modulated-signal sidebands are characterized by the regular (smooth) envelope of the pulse-modulated signal and conditions which would result in sideband resolution with pulse signals yield only

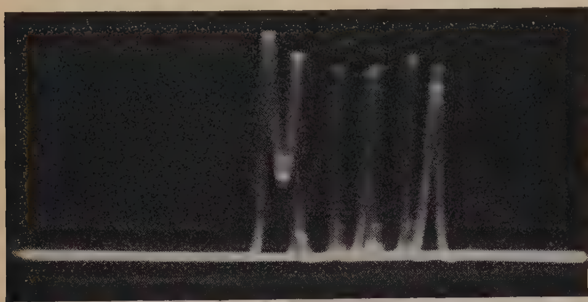

Fig. 7—Frequency-modulation signal, 35-kilocycle deviation, 2000-cycle modulation; $F=300$ kilocycles; $S=10$ kilocycles; $T=1/40$ second.

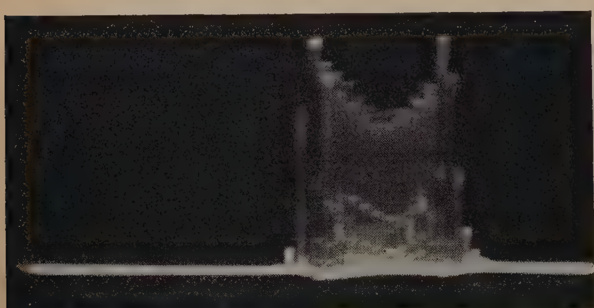

Fig. 8—Frequency-modulation signal, 50-kilocycle deviation, 150-cycle modulation; $F=300$ kilocycles; $S=10$ kilocycles; $T=1/40$ second.

"quasi" or smoothed versions of the true envelope unless the sidebands are also resolved.

Experimentally it has been observed that, if the condition (1) is met, both amplitude- and angular-modulated signals will show clear sideband structures. Figs. 5 and 6 show such frequency-modulated and amplitude-modulated signals. If the resolving power is sufficiently coarse to include the signal and all its sidebands, frequency-modulated signals will appear as "pips" of constant amplitude, oscillating in position. Similarly amplitude-modulated signals will appear as "pips" constant in position and oscillating in amplitude.

For resolving power intermediate between the above and that of (1) frequency-modulated signals show a number of oscillating "pips" with a quasi envelope as in Figs. 7 and 8. Amplitude-modulated signals show a single "pip" serrated in envelope as in Fig. 9. These serrations represent the changes in amplitude caused by modulation taking place during the time the signal is in the pass band of the analyzer.

Fig. 9—Amplitude-modulation signal, 16-kilocycle, modulation 40 per cent; $F=100$ kilocycles; $S=10$ kilocycles; $T=1/40$ second.

Limitations of Spectrum Analyzers

It is feasible to design recording spectrum analyzers for any desired resolving power, since the sweep rate may be made as low as necessary. Spectrum analyzers with cathode-ray-tube presentation are limited to sweep rates which permit reasonable visual persistence and fall into two classes.

1. Narrow-band (not more than 100 kilocycles) devices tracing true signal-sideband structure of signals modulated at audible or higher rates.

2. Devices scanning bands 1 megacycle or more in width. These are capable only of showing signal sidebands with signal modulation frequencies substantially higher than voice frequencies, and therefore cannot be relied upon for more than an indication of signal frequency, although in specific instances sideband resolution or envelope resolution may occur.

Principal and Complementary Waves in Antennas*

S. A. SCHELKUNOFF†, FELLOW, I.R.E.

Summary—In response to an increased interest in mathematical aspects of antenna theory, this paper presents details of analysis of cylindrical and other nonconical antennas as a supplement to a previous paper[1] containing the outline of the method and the main results. In the course of the present discussion the theory of principal waves on cylindrical conductors is extended to include the case in which the diameter is not small compared with the wavelength.

Introduction

NOT VERY long ago, Dr. L. Brillouin and I spent some time discussing the antenna theory and the discrepancies between impedance values obtained[2] from a solution of the Oseen-Hallén approximate integral equation and from a direct approximation to the solution of Maxwell's equations.[1] The discrepancies are explained in the companion paper[3] where it is shown that the approximations involved in the integral equation are justified, that insufficient accuracy of Hallén's first approximation to the solution leads to a degradation of subsequent approximations, and that the revised procedure[4] employed by Miss Marion C. Gray should and does lead to a better series. It is difficult to overemphasize the importance of a proper choice of the fundamental parameter in the reciprocal powers of which one is naturally led to expand the current distribution in the antenna. This parameter is not uniquely defined by the mathematical equations and its choice controls the goodness of the approximation consisting of only the first two or three terms of the expansion.

In order to complete our discussion of the fundamentals of antenna theory, this paper presents mathematical details of the other analysis of cylindrical antennas which is based on representation of the field around the antenna in terms of appropriate solutions of Maxwell's equations. The present paper should be regarded as a supplement to the paper[1] already referred to, which contains a suggestive outline of the method, actual results, and their interpretation, for antennas of several shapes, but treats in detail only conical antennas. Besides yielding a solution of the antenna problem, this method leads to an attractive physical picture of the phenomenon of radiation and focuses one's attention on similarities as well as dissimilarities between antennas and transmission lines. It should be stressed that this analogy can be made *a posteriori*, after the nature of the solution has been examined in the light of Maxwell's equations, and should not be confused with *a priori* assumptions of the analogy in some earlier work. In this early work an intuitive analogy had been made between antennas and ordinary idealized transmission lines admitting *only one* transmission mode; but it was subsequently discovered[1] that antennas can be regarded only as transmission lines with several modes of transmission of which, however, one, the "principal" mode, dominates the rest.

Structure of the Solution

In contrast with the method employed by Oseen and Hallén,[5] in which attention is concentrated at once on the current in the antenna, our method depends on the analysis of the field around the antenna and subsequent determination of the current associated with this field. The analysis is carried out in spherical co-ordinates, and for this reason the space is divided into the antenna region (1) and the remainder (2) with a spherical boundary between the two, Fig. 1. The reason for subdivision is that the boundary conditions on the axis of the antenna $\theta=0$, π are different for the two regions. In the external region (2) the solution should not be singular on the axis; this condition leads to a possibility of expressing the most general field in region (2) as a series of *integral* spherical harmonics. On the other hand, in

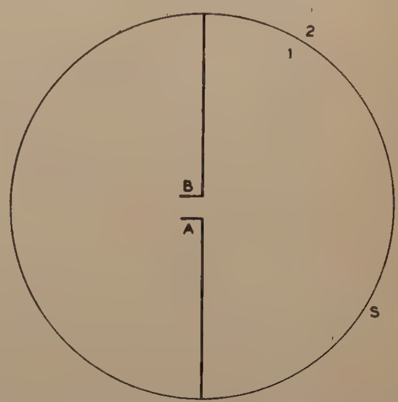

Fig. 1—Division of space into the antenna region (1) and the external region (2); S is the sphere centered at the input terminals and passing through the ends of the antenna.

* Decimal classification: R120. Original manuscript received by the Institute, June 18, 1945; revised manuscript received, August 20, 1945.

† Bell Telephone Laboratories, New York, N. Y.

[1] S. A. Schelkunoff, "Theory of antennas of arbitrary size and shape," Proc. I.R.E., vol. 29, pp. 493–521; September, 1941.

[2] Ronold King and F. G. Blake, Jr., "The self-impedance of a symmetrical antenna," Proc. I.R.E., vol. 30, pp. 335–349; July, 1942.

[3] S. A. Schelkunoff, "Concerning Hallén's integral equation for cylindrical antennas," Proc. I.R.E., vol. 33, pp. 872–878; December, 1945.

[4] Marion C. Gray, "A modification of Hallén's solution of the antenna problem," *Jour. Appl. Phys.*, vol. 15, pp. 61–65; January, 1944.

[5] E. Hallén, "Theoretical investigations into the transmitting and receiving qualities of antennas," *Nova Acta Uppsala*, ser. IV, vol. 11, pp. 1–44; November, 1938.

region (1) the axis is excluded by the antenna and although the solution is permitted to have singularities on the axis, it is required to satisfy certain boundary conditions on the surface of the antenna. This leads to a representation of the solution in terms of *fractional* spherical harmonics. The method is particularly suitable to biconical antennas. For other shapes, the method is still practicable when the transverse dimensions of the antenna are small, in which case the antenna becomes a "cone with slowly varying angle", and the solution may be expressed as a series of "perturbed" fractional spherical harmonics.

Having expressed the solutions in the regions (1) and (2) as series with arbitrary coefficients, we find that the requirement of continuity of the field at the boundary sphere S furnishes enough equations for determination of all these coefficients except one. This last unknown is expressed in terms of the impressed voltage.

The next step in the breakdown of the field into component parts is a representation of the field in the antenna region as the sum of the principal and complementary waves. The simplest way to explain the nature of the principal waves is to say that these are the waves which would be generated in an infinitely long antenna. An outward-moving wave is generated by the source at A, B and an inward-moving wave could be generated by reflection from a conducting sphere concentric with A, B. These are the waves in which electric lines run substantially along the meridians, Fig. 2; exactly along

Fig. 2—Electric lines of force in principal waves.

the meridians for the biconical antenna. These are the waves which are exactly transverse electromagnetic waves in the case of the biconical antenna and very nearly transverse electromagnetic in other cases. These are the waves which correspond to the well-known waves along parallel wires, coaxial cylinders, and other "two-conductor transmission lines."

For a finite antenna the field, consisting of principal waves alone, will not satisfy the continuity requirements at the boundary sphere S since in region (2) there is no principal wave to match. In fact, all waves in region (*2*) have a *radial* electric intensity and the condition of continuity is satisfied by adding a proper *complementary* field in region (1) which also possesses a radial electric intensity. This added field is required, therefore, in consequence of the sudden termination of the wires. Its presence expresses the fact that the reflection of the boundary sphere is not uniform, for otherwise we should have had merely a principal reflected wave (as when the sphere is a perfect conductor). At first it may seem strange that we should speak of reflection from a purely geometric boundary. We could dispose of it as a peculiarity of the co-ordinate system we are using, as an attribute of mathematics rather than as a disclosure of the underlying physical reality; but there is more to it than this. If we apply a voltage across A, B for a very brief interval of time, a thin spherical electromagnetic bubble is generated. The bubble will expand outwards and the mechanism of its expansion is given by Huygens' principle or its more complete form known as the induction theorem.[6] This simple movement persists until the forward boundary of the bubble reaches the end of the antenna, when the disturbance becomes "aware of" the altered conditions ahead. Naturally, this awareness manifests itself first near the wire and there the reflection is the greatest. It is in this respect that the present case differs from that of a uniform change in the characteristics of the medium over the entire wave front.

Thus the principal feature of the method is: waves in infinitely long antennas are considered first; subsequently, the complementary waves are included to express the effect of sudden termination of the wires, particularly with regard to uneven reflection at the wave front passing through the end of the antenna.

All this constitutes the background both for the subsequent mathematical analysis and for the physical interpretation of the results. For further ideas on this subject, the reader is referred[7,8] to the literature.

Principal Waves

In everything that follows, we assume perfect conductors and dielectrics because our main concern is radiation. For a double cone, Fig. 3, it is easy to find the exact solution of Maxwell's equations for principal waves; thus

$$H_\phi^+ = \frac{I^+ e^{-i\beta r}}{2\pi r \sin\theta}, \quad E_\theta^+ = \frac{60 I^+ e^{-i\beta r}}{r \sin\theta}, \quad \beta = 2\pi/\lambda,$$

[6] S. A. Schelkunoff, "Electromagnetic Waves," D. Van Nostrand Co., Inc., New York, N. Y., 1943, p. 399.

[7] J. C. Slater, "Microwave Transmission," McGraw-Hill Book Company, New York, N. Y., 1941, pp. 219–232.

[8] Simon Ramo and John R. Whinnery, "Fields and Waves in Modern Radio," John Wiley and Sons, Inc., New York, N. Y., 1944, pp. 482–491.

$$H_\phi^- = \frac{I^- e^{i\beta r}}{2\pi r \sin\theta}, \quad E_\theta^- = -\frac{60 I^- e^{i\beta r}}{r \sin\theta}; \tag{1}$$

the remaining components of the field are equal to zero. In the antenna theory we shall be concerned with the *transverse voltage*, $V(r)$; that is, the line integral of E along a typical meridian

$$V(r) = \int_{\psi}^{\pi-\psi} r E_\theta d\theta, \tag{2}$$

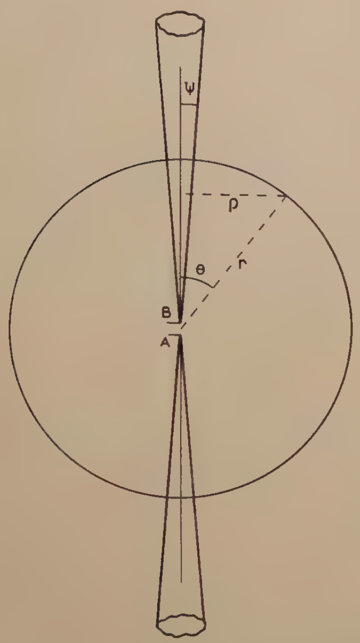

Fig. 3—A biconical antenna.

and with the current $I(r)$ in the upper cone,

$$I(r) = 2\pi r \sin\theta H_\phi = 2\pi r \sin\psi H_\phi. \tag{3}$$

If we write the expressions for V and I, we shall find that they are exactly the same as for a uniform transmission line with the following characteristic impedance:

$$K = 120 \log \cot(\psi/2) \simeq 120 \log(2/\psi). \tag{4}$$

The approximation is for small values of the cone angle ψ.

In fact, if we introduce into Maxwell's equations our definition of the principal wave (in the present case $E_r = E_\phi = H_r = H_\theta = 0$) and V and I from (2) and (3) in the place of E_θ and H_ϕ, the following equations are obtained after suitable integrations

$$\frac{dV}{dr} = -i\omega LI, \quad \frac{dI}{dr} = -i\omega CV, \tag{5}$$

$$L = (\mu/\pi) \log \cot(\psi/2), \quad C = \pi\epsilon/\log \cot(\psi/2). \tag{6}$$

Equations (5) hold, in fact, for principal waves on any pair of coaxial cones, Fig. 4, or even for any pair of cones with a common apex, Fig. 5; only the values of L and C are different. If we remove the common apex of the cones to infinity, we shall find that a coaxial pair of cylinders is a limiting case of coaxial cones and the pair of parallel wires is a limiting case of diverging cones shown in Fig. 5.

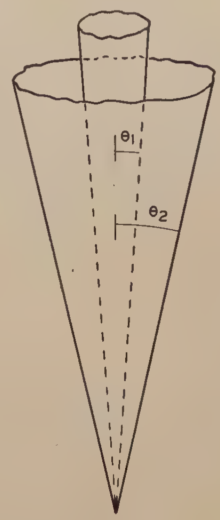

Fig. 4—Coaxial cones with a common apex.

It should be noted that the transverse voltage defined by (2) is equal not to the difference of scalar retarded potentials at the ends of the corresponding meridian but

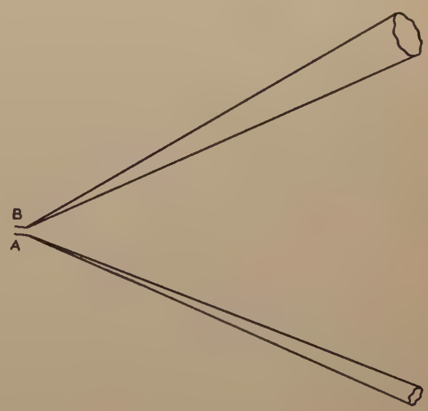

Fig. 5—Cones with a common apex.

to the difference of scalar electric potentials of the kind which appear in the theory of spherical waves.[6] In this theory the retarded potentials would be very cumbersome and for this reason are not used.

The field intensities (1) become infinite as r approaches zero; but V and I remain finite. In this respect the principal waves differ from the complementary waves. Infinite voltages are required for generation of *progressive* complementary waves by a point source and

very large voltages in the case of a source of finite but small dimensions. We shall return to this topic in the next section.

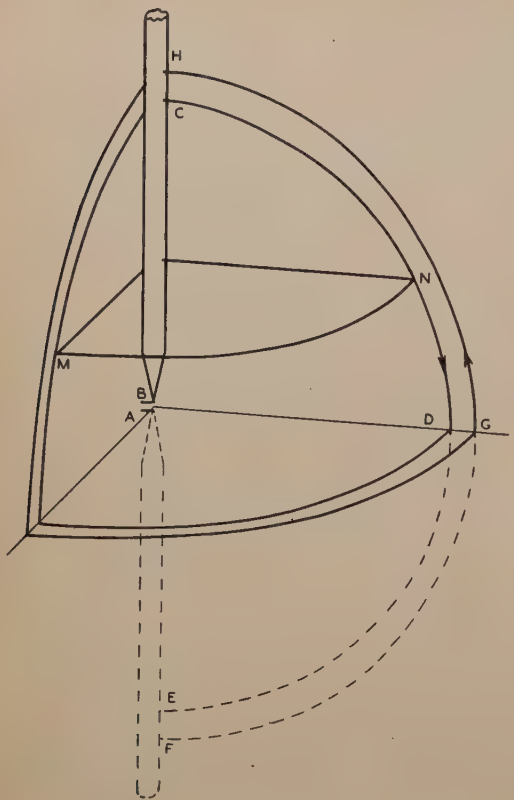

Fig. 6—A cylindrical antenna with tapered input terminals.

If the conductors are nonconical but of such proportions (Fig. 6) that the electric lines should nearly coincide with the meridians, we expect that equations (5) and (6) will be nearly correct if we assume that the cone angle ψ is varying continuously with r; thus

$$\psi = \sin^{-1}(a/r), \tag{7}$$

where a is the radius of the conductor where it is intercepted by the sphere of radius r. While this is practically obvious, some questions may be raised unless a few details are supplied. We can start either with Maxwell's differential equations or apply the fundamental laws directly. Thus, applying Faraday's law of the electromotive force to the curvilinear rectangle $CDEFGHC$, Fig. 6, we have

$$\frac{dV}{dr} = -i\omega\mu \int_{\psi}^{\pi-\psi} H_{\phi} r d\theta. \tag{8}$$

Applying Ampère-Maxwell's law of the magnetomotive force to a typical magnetic line MN, we obtain

$$H_{\phi} = \frac{I(r) + 2\pi i\omega\epsilon r^2 \int_{\psi}^{\theta} E_r \sin\theta d\theta}{2\pi r \sin\theta}. \tag{9}$$

This equation expresses the equality of the magnetomotive force $(2\pi r \sin\theta)H_{\phi}$ round the magnetic line and the total radial electric current through the spherical segment bounded by this line. The first term $I(r)$ is the conduction current in the upper conductor at the place where it cuts the sphere and the second term is the radial displacement current. If we neglect the latter, which we are entitled to do in the case of principal waves, we shall obtain the leading equations in the sets (5) and (6). Substituting from (9) into (8) we have

$$\frac{dV}{dr} = -i\omega LI + \beta^2 r^2 \int_{\psi}^{\pi-\psi} \frac{d\theta}{\sin\theta} \int_{\psi}^{\theta} E_r \sin\theta d\theta. \tag{10}$$

This is the exact equation.

In order to obtain the second transmission equation, we start with the following equation from Maxwell's set:

$$-i\omega\epsilon r E_{\theta} = \frac{\partial}{\partial r}(rH_{\phi}) \tag{11}$$

and substitute from (9); thus

$$-i\omega\epsilon r E_{\theta} = \frac{1}{2\pi \sin\theta} \frac{dI}{dr} + \frac{\partial}{\partial r}\left[\frac{i\omega\epsilon r^2}{\sin\theta}\int_{\psi}^{\theta} E_r \sin\theta d\theta\right]. \tag{12}$$

Integrating from $\theta = \psi$ to $\theta = \pi - \psi$ and substituting from (2), we have

$$-i\omega\epsilon V = \left(\frac{1}{\pi}\log\cot\frac{\psi}{2}\right)\frac{dI}{dr} + \frac{d}{dr}\int_{\psi}^{\pi-\psi} \frac{i\omega\epsilon r^2}{\sin\theta} d\theta \int_{\psi}^{\theta} E_r \sin\theta d\theta. \tag{13}$$

Finally dividing by the coefficient of dI/dr, we obtain

$$\frac{dI}{dr} = -i\omega CV - i\omega C\frac{d}{dr}\int_{\psi}^{\pi-\psi} \frac{r^2 d\theta}{\sin\theta}\int_{\psi}^{\theta} E_r \sin\theta d\theta. \tag{14}$$

This time the exact equation contains the derivative of the correction term in (10).

Since the application of this equation is contemplated only when ψ is small, $\cot(\psi/2) \simeq 2/\psi \simeq 2r/a(r)$ where $a(r)$ is the radius of the conductor; furthermore $r \simeq z$ so that (6) becomes

$$L = \frac{\mu}{\pi}\log\frac{2r}{a(r)} = \frac{\mu}{\pi}\log\frac{2z}{a(z)},$$

$$C = \frac{\pi\epsilon}{\log[2r/a(r)]} = \frac{\pi\epsilon}{\log[2z/a(z)]}. \tag{15}$$

The last terms in (10) and (14), which we will neglect, are at least of the order of the square of the radius while the remaining terms depend upon the logarithm of the radius.

The other extreme arises in the case of cylindrical antennas, Fig. 7, or other antennas of revolution, in the immediate vicinity of a *line source*, MN. As the radius of the line source increases, a wedge is approached, Fig. 8. The voltage is applied between the edges AC

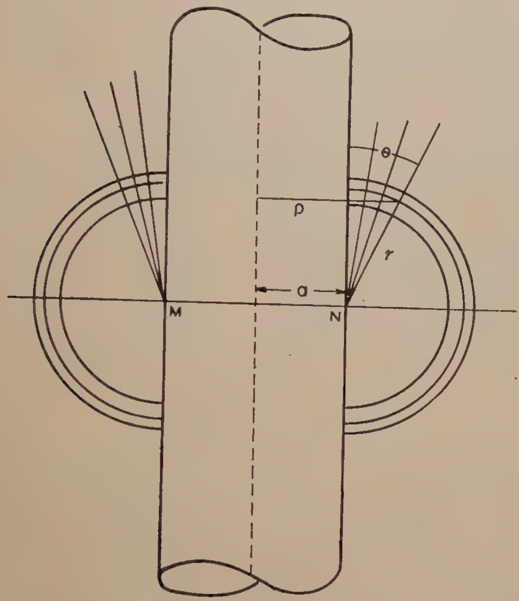

Fig. 7—A co-ordinate system suitable for cylindrical conductors.

and BD (which are assumed to be very close). In this case we still have equations (5) with r being the distance from the line halfway between the edges; the values of L and C per unit length along AC are

$$L = \pi\mu r \qquad C = \epsilon/\pi r. \tag{16}$$

The intermediate case presents the greatest mathematical difficulties. The most natural co-ordinate system would be that formed by the equipotential surfaces, with the two conductors being kept at constant potentials, and two orthogonal families of surfaces passing through the lines of electric force; then we should try to express the relationship between the voltage along the electric line of force and the current in the form analogous to (10) and (14), with L and C having their static values and residual terms when required. Unfortunately the equations become quite complicated.

In the case of cylindrical conductors, we have an alternative which possesses certain advantages, Fig. 7. One of the co-ordinates r is taken to be the distance from the "origin circle" MN; half planes issuing from the axis of the cylinder are designated by the azimuth angle ϕ as in spherical and cylindrical co-ordinates; and θ is the angle made by the generators of the cylinder with a typical radius in a ϕ plane. In these co-ordinates the field equations become

$$E_r = \frac{P}{2\pi i\omega\epsilon r^2}\frac{\partial\Psi}{\partial\theta}, \qquad E_\theta = -\frac{P}{2\pi i\omega\epsilon r}\frac{\partial\Psi}{\partial r},$$

$$H_\phi = \frac{\Psi}{2\pi\rho} = \frac{P\Psi}{2\pi r}, \qquad P = \frac{r}{a + r\sin\theta}, \tag{17}$$

$$\frac{\partial}{\partial r}\left(P\frac{\partial\Psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(P\frac{\partial\Psi}{\partial\theta}\right) = -\beta^2 P\Psi.$$

From the last equation we obtain

$$\frac{\partial^2\Psi}{\partial r^2} + \beta^2\Psi = -\frac{\partial}{\partial r}(\log P)\frac{\partial\Psi}{\partial r} - \frac{1}{r^2}\frac{\partial^2\Psi}{\partial\theta^2} - \frac{1}{r^2}\frac{\partial}{\partial\theta}(\log P)\frac{\partial\Psi}{\partial\theta}, \tag{18}$$

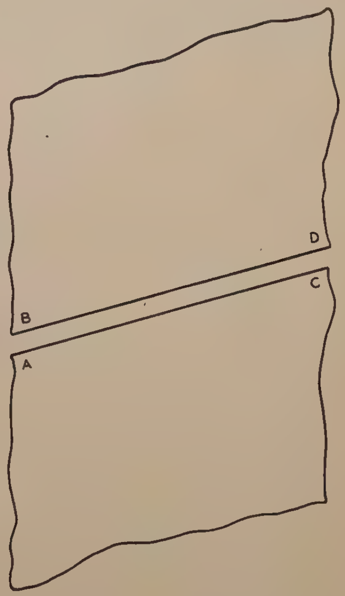

Fig. 8—A portion of a biplanar radiator.

where

$$\frac{\partial}{\partial r}(\log P) = \frac{a}{r(a + r\sin\theta)}, \qquad \frac{\partial}{\partial\theta}(\log P) = -\frac{r\cos\theta}{a + r\sin\theta}. \tag{19}$$

The solution for principal waves is the one in which Ψ is nearly independent of θ so that E_r is nearly zero everywhere. In the first approximation we ignore the last two terms in (18) and obtain

$$\frac{\partial^2\Psi}{\partial r^2} + \beta^2\Psi = -\frac{a}{r(a + r\sin\theta)}\frac{\partial\Psi}{\partial r}. \tag{20}$$

If r is small compared with a, then

$$\frac{\partial^2\Psi}{\partial r^2} + \beta^2\Psi = -\frac{1}{r}\frac{\partial\Psi}{\partial r}; \tag{21}$$

in this case Ψ is a Bessel function of order zero and the field is nearly that for a wedge formed by two half planes. If r is large compared with a, the right-hand side of (20) can be neglected and Ψ is an exponential function. In this case, the field is given by expressions of the type (1) and equations (5) and (15) apply even if a is large. The function Ψ is substantially independent of θ when r is very much smaller than or very much greater than a; the greatest variation with θ occurs in the vicinity of $r=a$ where the coefficient of the last term in (20) varies from $(-1/a)$ at $\theta=0, \pi$ to $(-1/2a)$ at $\theta=\pi/2$.

We are now in a position to determine the values of L and C for principal waves on a cylinder of any radius, except for the corrections for a small electric intensity in the direction of wave propagation. Thus in the present case equation (8) becomes

$$\frac{dV}{dr} = -i\omega\mu\int_0^\pi rH_\phi d\theta. \tag{22}$$

On the other hand, $\Psi=I$ and substituting from (17) we have

$$\frac{dV}{dr} = -\frac{i\omega\mu I}{2\pi}\int_0^\pi \frac{rd\theta}{a+r\sin\theta}; \tag{23}$$

therefore

$$L = \frac{\mu}{2\pi}\int_0^\pi \frac{rd\theta}{a+r\sin\theta}. \tag{24}$$

Similarly, C is obtained from an equation corresponding to (2) and from (17). It turns out that $LC=\mu\epsilon$. Carrying out the required integration, we find that in free space

$$\begin{aligned}\sqrt{L/C} &= \frac{240r/a}{\sqrt{1-(r^2/a^2)}}\tan^{-1}\sqrt{\frac{a-r}{a+r}}, && r<a,\\ &=120, && r=a,\\ &=\frac{240r/a}{\sqrt{(r^2/a^2)-1}}\log\frac{\sqrt{r+a}+\sqrt{r-a}}{\sqrt{2a}}, && r>a,\\ &=\frac{240r/a}{\sqrt{(r^2/a^2)-1}}\tanh^{-1}\sqrt{\frac{r-a}{r+a}}, && r>a.\end{aligned} \tag{25}$$

In a dielectric medium other than vacuum, we should multiply (25) by $\sqrt{\mu/\epsilon}/120\pi$. Fig. 9 represents $\sqrt{L/C}$ as a function of r/a in the intermediate region where the simple formulas (15) and (16) are inapplicable. The dotted curves show the behavior of the simple formulas in this region.

The integrals of $\sqrt{L/C}$ play an important role in the theory. These integrals are

$$\begin{aligned}&\int_r^a \sqrt{L/C}\,dr\\ &=240a\left[\sqrt{1-(r/a)^2}\,\tan^{-1}\sqrt{\frac{a-r}{a+r}}-\frac{a-r}{2a}\right], r<a;\\ &\int_a^r \sqrt{L/C}\,dr\\ &=240a\left[\sqrt{(r/a)^2-1}\,\tanh^{-1}\sqrt{\frac{r-a}{r+a}}-\frac{r-a}{2a}\right], r>a.\end{aligned} \tag{26}$$

We are also in a position to appraise the magnitude of the error we make when we neglect the radial electric intensity, that is, the last term in (10). At first sight, it may appear that the error increases with r; but the reference to equations (17) and (20) shows that the error decreases with r when r is fairly large[9] compared with a.

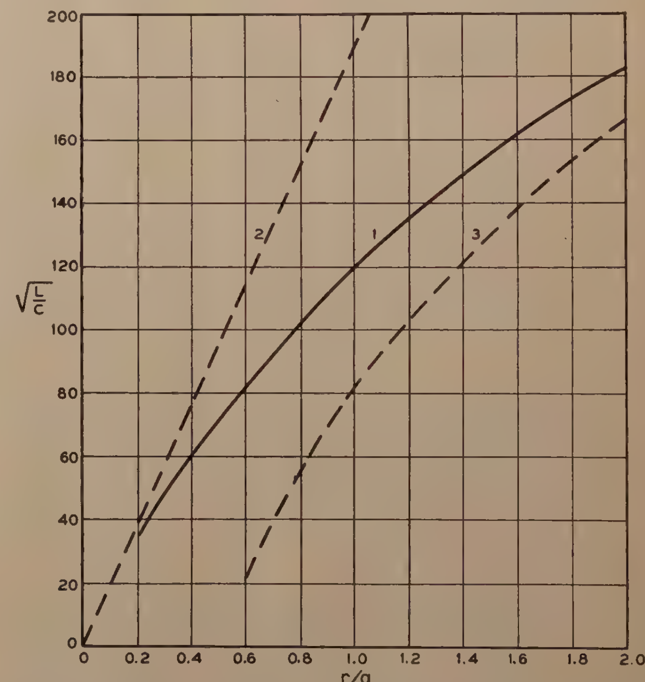

Fig. 9—The nominal characteristic impedance ($\sqrt{L/C}$) as a function of r/a (see Fig. 7). Curve (1) represents equation (25); curve (2) represents $\sqrt{L/C}=60\pi r/a$, an approximation when r is small compared with a; curve (3) represents $\sqrt{L/C}=120\log(2r/a)$, an approximation when r is large compared with a.

Thus if $\theta\neq 0, \pi$, the magnitude of E_r varies inversely as r^{-4}; if $\theta=0, \pi$, E_r seems to vary as r^{-3} but then it is actually equal to zero because of the boundary. Hence the last term in (10) varies as r^{-2} or more precisely as $a\lambda/r^2$.

The spherical system of co-ordinates and the system shown in Fig. 7 seem to be the most suitable co-ordinates in the solution of the antenna problem by the present

[9] In this case there is little difference between r as defined in Fig. 7 and the distance r from the center of the spherical co-ordinate system employed in (10).

method. One might suppose that spheroidal co-ordinates would be particularly suited to spheroidal conductors; but this does not happen to be the case. In the first place, the spheroidal functions receiving the greatest attention in the literature correspond to certain particular distributions of the impressed voltage over the entire spheroid. These special solutions correspond to natural radial modes of propagation rather than to traveling waves on the spheroid. The required solution for a concentrated source is then constructed from these special solutions and the analysis resembles that usually employed in the problem of the vibrating string. On the other hand, the present method is based on traveling waves and is analogous to that usually employed in the transmission-line theory. Naturally, there must exist spheroidal functions to represent traveling waves; but their theory has not been developed as yet. The needed functions are those solutions of

$$(1 - u^2)\frac{d^2M}{du^2} + (k^2 - \omega^2\mu\epsilon l^2u^2)M = 0 \qquad (27)$$

which are singular at $u = \pm 1$; they do not promise to be particularly simple and on the whole the spherical co-ordinates seem to be more suitable for thin spheroids treated by the present method.

Complementary Waves

In a sense, principal waves belong to the conductors, since nothing quite like them exists without the conductors. A generator of infinitesimal size imbedded in a homogeneous dielectric medium will produce no field if the electromotive force is finite. We must have a conducting wire connected to each terminal before we can hope to create a finite field and the principal waves are the waves that make the difference.

A generator of finite size creates waves in a perfectly homogeneous medium and these waves are merely modified when conductors are connected to the terminals of the generator. Such waves may be said to belong to the medium. Consider, for example, circularly symmetric fields, and suppose that θ is the angle made by a typical radius with the axis of symmetry. If the medium is homogeneous, the dependence of the field on θ is represented by the Legendre function $P_n(\cos\theta)$, where n is a positive integer. If we place thin wires along the axis, then n becomes a fraction such that the difference from an integer approaches zero with the reciprocal of the logarithm of the radius of the wire.

For a biconical antenna such "complementary" waves are expressed quite simply in terms of spherical co-ordinates. The field intensities satisfy the following equations:

$$\frac{\partial}{\partial r}(rE_\theta) = -i\omega\mu(rH_\phi) + \frac{\partial E_r}{\partial\theta}, \qquad (28)$$

$$\frac{\partial}{\partial r}(rH_\phi) = -i\omega\epsilon(rE_\theta), \qquad (29)$$

$$E_r = \frac{1}{i\omega\epsilon r^2\sin\theta}\frac{\partial}{\partial\theta}[\sin\theta(rH_\phi)], \qquad (30)$$

$$\frac{\partial^2}{\partial r^2}(r^2E_r) = -\omega^2\mu\epsilon(r^2E_r) - \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta\,\frac{\partial(r^2E_r)}{\partial\theta}\right]. \qquad (31)$$

Equation (31) possesses one simple solution, namely $E_r = 0$; this yields the principal waves. The complementary waves are defined by

$$r^2E_r = R(r)\Theta(\theta), \qquad (32)$$

where at the surface of the cone

$$\Theta(\psi) = \Theta(\pi - \psi) = 0. \qquad (33)$$

In particular we are interested in the case in which $\Theta(\theta)$ satisfies the following condition

$$\Theta(\pi - \theta) = -\Theta(\theta), \qquad (34)$$

corresponding to the symmetrical current distribution in the antenna. It turns out[1] that

$$\Theta(\theta) = \tfrac{1}{2}[P_\nu(\cos\theta) - P_\nu(-\cos\theta)], \qquad (35)$$

where for small values of ψ

$$\nu = 2n + 1 + \frac{1}{\log(2/\psi)} = 2n + 1 + \frac{120}{K}, \qquad n = 0, 1, 2, \cdots. \qquad (36)$$

The dependence of the field on the radial co-ordinate is expressed by

$$\frac{\partial}{\partial r}(rE_\theta) = -\left[i\omega\mu + \frac{\nu(\nu+1)}{i\omega\epsilon r^2}\right](rH_\phi), \qquad (37)$$

$$\frac{\partial}{\partial r}(rH_\phi) = -i\omega\epsilon(rE_\theta).$$

The solutions of these equations can be expressed in terms of Bessel functions.

For other than biconical antennas, we allow ψ and therefore ν to be a function of r

$$\psi = a(r)/r, \qquad \nu = 2n + 1 + 1/\log[2r/a(r)]. \qquad (38)$$

We now have all the functions needed for representation of the field within the antenna region, and our next task is to match this field to the field outside this region.

Matching of Waves at the Boundary Sphere

There are at least two methods for matching the fields at the boundary sphere S of regions (1) and (2), Fig. 1.

One is the method of successive approximations based on the following considerations:

1. In the first approximation we neglect the complementary waves. When taken together with the vanishing of current at the ends of the antenna, this implies also the vanishing of the magnetic intensity over the boundary sphere. This boundary condition leads to an equation for two arbitrary constants in the general expression for principal waves. The second equation is obtained from the boundary condition at the generator where the impressed voltage is supposed to be given. Thus we obtain a field which satisfies all the boundary conditions in region (1) but which is discontinuous at S. This step yields strictly sinusoidal current distributions for biconical antennas, and nearly sinusoidal distributions for antennas of other shapes.

2. The next step is to determine the field of the above-found current distribution by the retarded potential method. In this way we obtain a field which is continuous at the boundary sphere S and which fails to satisfy the boundary conditions at the surface of the antenna because the tangential component of the electric intensity will not vanish there. This is a well-known fact which has caused a great deal of uneasiness in the past. It should not have, since no approximation can possibly satisfy *all* requirements of the problem; if it did, it would be the exact solution. The important thing is the magnitude of the error and not merely an indication that there is an error.

3. Having carried out the second step, we expand one of the field components at the boundary sphere in terms of wave functions appropriate to region (1). This will insure that the vanishing of the tangential electric intensity will be re-established at the surface of the antenna; but the continuity conditions at S will be broken automatically. The new field in the antenna region yields a new current distribution in the antenna.

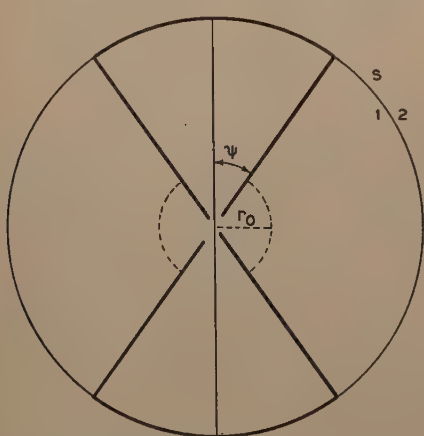

Fig. 10—A biconical antenna of large angle Ψ.

Steps (2) and (3) are then repeated *ad infinitum*. In the original antenna paper, the process was continued with step (3). There, besides the principal current, we found the term inversely proportional to the average characteristic impedance of the antenna. The next two steps would have yielded a term varying inversely as the square of the average characteristic impedance.

Another method leads to a system of linear algebraic equations with an infinite number of unknowns. In theory the method is very simple. In region (1) the field is expressed as an infinite series of wave functions satisfying the boundary conditions on the antenna; in region (2) the field is expressed as an infinite series of spherical harmonics of integral degree; and the conditions at the boundary between the two regions furnish the necessary equations for calculation of the coefficients in the series. Thus in region (1) we have

$$\begin{aligned} rH_\phi &= a_0 f_0^+(r,\theta) + b_0 f_0^-(r,\theta) + \sum_{n=1}^{\infty} a_n f_n(r,\theta), \\ i\omega\epsilon r E_\theta &= -a_0 \frac{\partial f_0^+(r,\theta)}{\partial r} - b_0 \frac{\partial f_0^-(r,\theta)}{\partial r} \\ &\quad - \sum_{n=1}^{\infty} a_n \frac{\partial f_n(r,\theta)}{\partial r}; \end{aligned} \tag{39}$$

and in region (2)

$$\begin{aligned} rH_\phi &= \sum_{n=0}^{\infty} A_n \widehat{K}_{2n+1}(i\beta r) P^1_{2n+1}(\cos\theta), \\ rE_\theta &= -\sum_{n=0}^{\infty} \sqrt{\mu/\epsilon}\, A_n \widehat{K}'_{2n+1}(i\beta r) P_{2n+1}^1(\cos\theta), \\ \widehat{K}_n(x) &= (2x/\pi)^{1/2} K_{n+1/2}(x). \end{aligned} \tag{40}$$

When $r=l$, H_ϕ and E_θ must be continuous. One way of expressing this condition is to equate (39) and (40) for a sequence of angles $\theta = k\pi/2^{m+1}$, $k=1, 2, 3, \cdots 2^m$, where $m \to \infty$. If the antenna has a conducting portion in the boundary sphere, Fig. 10, then over this portion E_θ as given by (40) should vanish. Perhaps, the best way of assuring this is to expand into spherical harmonics the function equal to E_θ as given by (39) over the nonconducting part of the boundary sphere and equal to zero over the rest of the sphere. The procedure is simplified by the fact that $P^1_{2m+1}(\cos\theta)$ form an orthogonal set of functions. In this way we express A_n's in terms of a_n's; and then we match H_ϕ. When the set of functions $f_n(r,\theta)$ is orthogonal, we can expand H_ϕ as given by (40) in the usual way and express a_n's in terms of A_n's. Thus linear homogeneous equations connecting a_n's are obtained. One linear nonhomogeneous equation is obtained from the condition at the source $(r=0)$: the integral $-\int rE_\theta d\theta$ should equal the impressed voltage.

In the above outline we have assumed that the source of power is infinitesimal. It is for this reason that we

Probe Error in Standing-Wave Detectors*

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Summary—Distorted patterns which are observed in standing-wave detectors with deeper probe penetrations are shown to be attributable to reflections at the probe wire. It is demonstrated that the probe, over a wide range of penetrations, acts as a simple shunt admittance across the transmission line.

The mathematical and graphical treatment developed on this basis gives a satisfactory account of observed probe patterns, and makes it possible to obtain exact readings even from badly distorted patterns. By applying the results, one is in a position to improve the sensitivity of standing-wave measurements at low-power levels without sacrifice in accuracy, simply by using much deeper probe penetrations.

I. Introduction

Reflections caused by the probe in conventional standing-wave detectors are not negligible even with probe penetrations of the order commonly used, and they may result in noticeable distortions of the observed standing-wave pattern. These distortions should serve as a warning not to take the apparent standing-wave ratio at its face value. But since the shape of the pattern rarely receives much attention, the effect is a potential source of error. It is, therefore, of some importance to have a simple criterion for its presence and a ready method of correction. The latter will be particularly desirable when it permits us to improve the sensitivity of measurements at very low power levels without sacrifice in accuracy, simply by using much deeper probe penetrations.

The method proposed here rests on the plausible assumption that the distortions are entirely attributable to reflections at the probe wire and that the probe, insofar as it causes reflections, may legitimately be considered as a mere shunt admittance across the line. The twofold purpose of the paper is, therefore (1) an experimental verification of these assumptions; and (2) their mathematical exploitation in the form of formulas and graphical procedure. From a purely practical viewpoint the second purpose is of greater interest, yet a large portion of the paper must be devoted to the experimental verification, showing the limitations of the procedure. In view of this duplicity of purpose, the group of readers primarily interested in the application would be best served, it is felt, by having a sample computation presented in Section II, while proofs and verifications are relegated to later sections. As shown in the sample, three galvanometer readings at specified probe positions suffice for a computation of the true standing-wave ratio even from a badly distorted pattern, and furthermore, of the admittance of the perturbing probe if desired. The determination of the latter is not actually required in routine measurements but will be used here as part of the experimental verification of the underlying assumptions.

Our experiments test the validity of these assumptions for a standing-wave detector operating in a rectangular wave guide at a wavelength of a few centimeters. In this typical case our formulas must be applied when the probe penetration exceeds 25 per cent, and they account perfectly for the observed distortions with penetrations as deep as 65 per cent. While the mathematical treatment offered here is not restricted to a particular type of transmission line, it must be realized that the specific values quoted are not directly comparable with probe penetrations in, let us say, a coaxial line where the deeper probe reaches into a stronger electric field. It is also necessary to point out that in our measurements the customary precautions for excluding all undesired frequencies were observed, and that failure to do so will cause additional distortions of a type not within the scope of our formulas.

In our measurements, the probe admittance is a function not only of the penetration but also of the tuning of the probe circuit by means of which the detector response was adjusted to a maximum value. Thus it would be almost impossible to compute the admittance, and all values quoted are empirical. However, by using two independent experimental methods for their determination, an additional check on the consistency of our theory was provided.

The most noticeable distortion of a typical pattern (Fig. 1) is the asymmetry of the maximum position relative to the minima, though this is not a necessary criterion for probe reflections. The pattern of Fig. 1 was taken with a probe penetrating 50 per cent of the guide height and with an actual power standing-wave ratio of 12.8 whereas the apparent value is only 11.0. Thus an error of 20 per cent could be incurred unless one is aware of the distortion. Also, the minimum is shifted from the true position which might cause an error in the angular position of the load point plotted in the Smith chart[1] by as much as 20 degrees.

Probably the simplest criterion for reflections is in terms of the separation between the minimum and each of what may be termed the two midpoints. By midpoint is meant a probe position such that the galvanometer reading C is related to the maximum and minimum readings, A and B, by the equation

$$C = \frac{2AB}{A + B}. \tag{1}$$

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[1] P. H. Smith, "Transmission-line calculator," *Electronics*, vol. 12, pp. 29–31; January, 1939.

In the absence of reflections, the probe response

$$P = \tfrac{1}{2}[(A+B) - (A-B)\cos\phi]$$

gives a sine pattern when plotted against the probe position ϕ (expressed in radians) and one verifies easily that the midpoints must be spaced at intervals

$$\eta = \pm \cos^{-1}\left[\frac{A-B}{A+B}\right]$$

from the minimum because this reduces P to the value (1). The last equation can, therefore, serve as an experimental criterion for the absence of probe reflections. In the presence of reflections, one determines the positions of maximum, minimum, and midpoints and expresses their spacings in radians, to a scale which renders the half wavelength equal to 2π. As shown in Appendix I, a mathematical relation between these four positions makes it possible to obtain a complete solution from the two known spacings between any three consecutive positions; for instance, between the minimum and the two midpoints. Still it will be necessary to determine the maximum value if not its position, since it enters into the definition of the midpoints. The exception where one can do completely without the maximum is

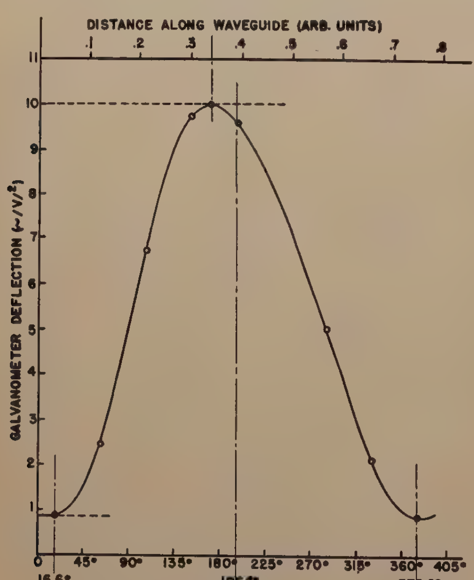

Fig. 1—Typical probe pattern. Points experimental, curves calculated.

the case of very high standing-wave ratios, where the midpoint response, according to (1), is simply twice the minimum response. This is fortunate because in this case the measurement of the maximum value would be impractical at power levels where the minimum reading has any accuracy. It turns out for very high standing-wave ratios that the probe reflections have little effect on the measured positions of minimum and midpoints because the correction terms are small of a high order.

II. Sample Calculations

Let the distorted pattern be the one shown in Fig. 1. The upper scale of abscissas represents actual distance along the wave guide in arbitrary units measured from an arbitrary position in the wave guide. The lower scale of abscissas represents angular distance along the wave guide in degrees and is based on 360 degrees between minima. We may compute the "midpoint" galvanometer deflection from (1)

$$C = \frac{2AB}{A+B} = \frac{2 \times 10.00 \times 0.87}{10.87} = 1.60$$

which enables us to read the midpoint positions from the illustration. From this illustration, therefore, we obtain the information contained in Table I.

Table I

	Maximum	Minimum 1	Minimum 2	Midpoint 1	Midpoint 2
Galvanometer Deflection	10.00	0.87	0.87	1.60	1.60
Angular Position (degrees)	170.0	16.6	376.6	46.6	343.0

Proceeding first by the graphical method, we draw an arbitrary circle C_1 (Fig. 2(a)), with center D, on which the angular positions from the table are marked. The

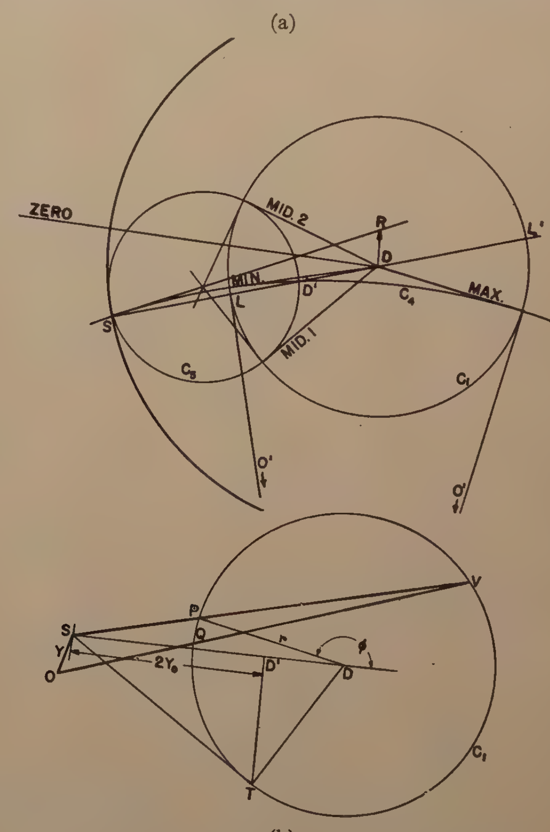

Fig. 2—Sample illustration of graphical procedure.

tangents at the maximum and minimum points intersect at O', the center of circle C_4 which is orthogonal to circle C_1. Similarly, the two tangents at the midpoint positions determine the center of circle C_5 which is orthogonal to circle C_1 and circle C_4. Through the outer intersection S of circle C_4 and C_5, draw the circle with center D. This circle is the outer rim of the Smith chart. The point L of intersection of the line SD with

the circle C_1, represents the true load point in the chart. This means that in the absence of probe reflections one would have found that the angular position of the minimum lay on the line SD when referred to the arbitrary zero of the angular scale as the axis of reals of the Smith chart, and that the ratio LD/SD is the true reflection coefficient.

This completes the operation necessary in a routine measurement. An equivalent analytical procedure consists in listing the angular spacings between the four characteristic points obtained from Table I as follows:

$$\alpha = 30 \text{ degrees}, \qquad \beta = 123 \text{ degrees},$$
$$\gamma = 174 \text{ degrees}, \qquad \delta = 33 \text{ degrees}.$$

From which we find

$$\epsilon = \frac{\beta - \alpha}{2} = 46.5 \text{ degrees}, \qquad \eta = \frac{\alpha + \beta}{2} = 76.5 \text{ degrees}$$

and

$$\sin \bar{\eta} = \frac{\sin \eta}{\sqrt{1 + \left(\dfrac{\sin \epsilon \cdot \sin \eta}{\cos \epsilon - \cos \eta}\right)^2}}$$
$$= \frac{0.972}{\sqrt{1 + \left(\dfrac{0.725 \times 0.972}{0.688 - 0.233}\right)^2}} = 0.519$$
$$\bar{\eta} = 31.27 \text{ degrees}.$$

From this we compute the reflection coefficient

$$r = \frac{1 - \tan \bar{\eta}/2}{1 + \tan \bar{\eta}/2} = \frac{1 - 0.280}{1 + 0.280} = 0.560$$

as compared with the apparent value 0.546.

In order to find the angular displacement μ of the minimum analytically, we use the equation

$$\sin (\eta - \mu) = \frac{\sin \eta}{\sqrt{1 + \cos^2 \eta \left(\dfrac{\cos \epsilon - \cos \eta}{\sin \epsilon \cdot \sin \eta}\right)^2}}.$$

Since the angle μ is quite small, this may be approximated by a simpler formula which actually gives better slide-rule accuracy.

$$\mu = \frac{\cos \eta \sin \eta}{2\left[\dfrac{1}{\sin^2 \bar{\eta}} - \dfrac{1}{\sin^2 \eta}\right]} = \frac{0.1165 \times 0.972}{3.75 - 1.06}$$
$$= 0.0422 \text{ radian}$$
$$= 2 \text{ degrees } 24 \text{ minutes}.$$

Since this angle is counted positive in the direction toward the nearer of the two midpoints, this will be found to check the graphical results previously given.

For completeness, we include in this section a determination of the probe admittance. Though this is not needed in a routine measurement, it permits us to generate the observed pattern graphically and thus provides a check on the whole procedure. Referring again to Fig. 2(a), a distance $(r/r') \times SD$ is laid off from S along the tangent at S to circle C_4, giving point R. The admittance Y is then represented by vector DR to the same scale to which $SD'/2$ represents the characteristic guide admittance Y_0. Numerically

$$Y = Y_0(0.45 + j0.06).$$

Referring to Fig. 2(b), we shall show how to generate a probe pattern given the load-reflection coefficient and

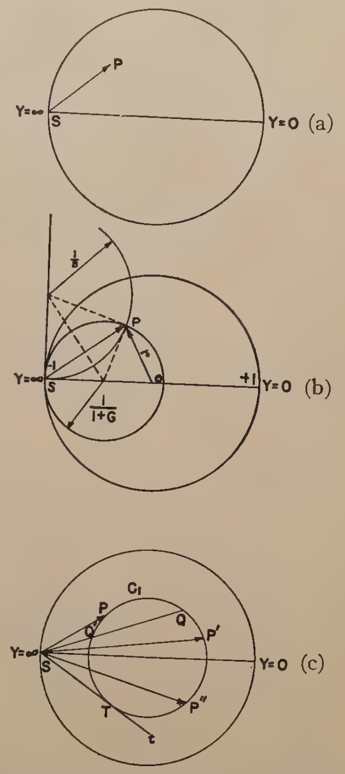

Fig. 3—Probe response as a vector in the Smith chart.

the probe admittance. Draw an arbitrary circle C_1 with center D. Construct the line SD so that the ratio r/SD is equal to the given reflection coefficient (here 0.56). From the tangent ST drop the perpendicular TD' to SD. Now to the scale of $SD'/2$ being equal to the guide admittance Y_0 construct the vector Y (probe admittance) to end on the point S. Here we have used the same Y as found from Fig. 2(a). Finally, draw any line SPV intersecting the circle and connect V and O. As P travels through angles ϕ the rectified galvanometer current varies as the square of the distance OQ. The probe pattern thus obtained is drawn as the curve of Fig. 1, the points of Fig. 1 being experimental. The proof of the construction of Fig. 2(b) will be found in Appendix III.

III. A Geometrical Presentation of Probe Response

It is possible to give a general presentation for the probe response by vectors in the Smith chart. Let P in Fig. 3(a) be the chart point representing a given load as it would be measured with a probe free of reflections. The angular position of P gives the position of a reference point relative to the position of minimum response, and the radius vector is a measure of the reflection coefficient. In particular, if P should fall at S (Fig. 3(a)

it would be indicative of zero probe response at the reference point, the reflection coefficient being unity with the minimum at the reference point. Now generally the voltage at the reference point is represented in phase and magnitude by the chart vector pointing from S to P, as can be proved in the following manner. We recall that[1] the contours of susceptance B are circles of radius $1/B$ (Fig. 3(b)) and the conductance circles have radii $1/(1+G)$. From the geometry of Fig. 3(b), we have

$$\sqrt{\left(\frac{1}{B}\right)^2-\left(\frac{SP}{2}\right)^2}+\sqrt{\left(\frac{1}{1+G}\right)^2-\left(\frac{SP}{2}\right)^2}$$

$$=\left(\sqrt{\left(\frac{1}{B}\right)^2+\left(\frac{1}{1+G}\right)^2}\right)$$

$$|\overrightarrow{SP}|^2=\frac{4}{B^2+(1+G)^2}.$$

To identify this with voltage, we will consider a line of characteristic admittance Y_0 terminated by Y. The ratio r of the incident A to the reflected C waves is

$$\frac{A}{C}=r=\frac{Y_0-Y}{Y_0+Y},$$

and the probe voltage V is

$$V=A+C=A\left(1+\frac{Y_0-Y}{Y_0+Y}\right)$$

$$|V|=\left|\frac{2AY_0}{Y_0+Y}\right|$$

and letting Y_0 and $A=1$, we have for unit incident amplitude

$$|V|^2=\frac{4}{|1+Y|^2}=\frac{4}{B^2+(1+G)^2}=|\overrightarrow{SP}|^2.$$

Thus the probe voltage may be represented by the length of the vector $\overrightarrow{SP}$. An alternative proof may be derived from first principles as follows:

Considering that the Smith chart is the complex plane of reflection coefficients r and that the short-circuit point S represents a reflection coefficient $r=-1$, (Y=infinity) we see that the vector $\overrightarrow{SP}$ represents the complex quantity $(1+r)$. On the other hand, the voltage at the reference point is the sum of the incident wave, of amplitude unity, and of the reflected wave, of amplitude and phase represented by r, and thus is given by the same complex quantity $(1+r)$.

At other reference points P', P'' $\cdots$ (Fig. 3(c)) the voltage is similarly given by the vectors SP', SP'', etc. Thus as we move the probe in a direction away from the load, the voltage vector $\overrightarrow{SP}$ varies as P rotates along the circle C_1 (Fig. 3(c)) in a clockwise sense, through angles which are proportional to the probe shifts. It is permissible here to ignore the relative position of probe and generator which affects merely the phase, not the magnitude of the probe voltage. The square of SP' is a measure of the rectified galvanometer current, the variation of which with probe position is a sine curve which may be seen by virtue of simple geometry.

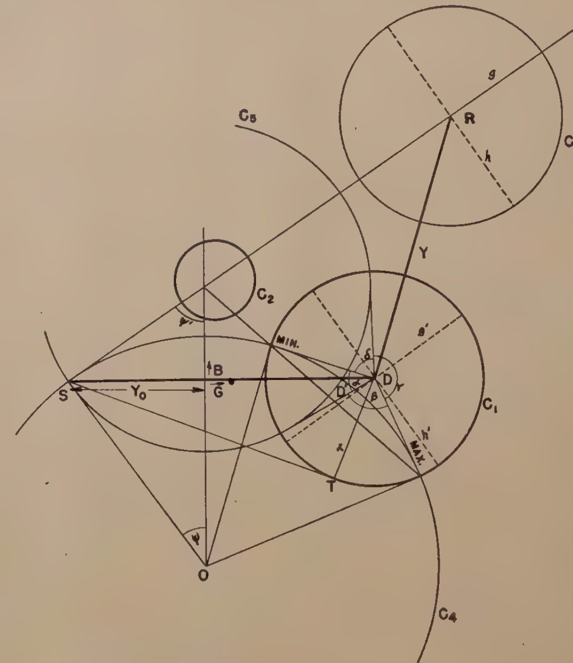

Fig. 4—Diagram representing general effect of probe admittance Y.

With a reflecting probe, circle C_1 no longer represents the load locus as seen at the probe but must be transformed, point by point, in a manner allowing for a constant additive amount of probe admittance. To find the result of this transformation, it is expedient to change temporarily from the chart diagram to the complex admittance plane. This transition is effected by means of an inversion; i.e., a transformation by reciprocal radii using the point S as the inversion center. It is expedient to choose the unit radius of the inversion in a manner such that circle C_1 as a whole remains unchanged by the transformation, its points merely changing places in pairs Q, Q' (See Fig. 3(c)). This is accomplished by choosing as the inversion unit the distance ST, T being the point of tangency of t and C_1 (Fig. 3(c)).

Referring now to Fig. 4, which represents a superposition of the chart and admittance diagrams, let C_1 and S have the same meaning as in Fig. 3(c). The distances between the four characteristic probe positions, expressed in radians, are designated as follows: α, the distance between the minimum and the nearest midpoint; β, between the maximum and its nearest midpoint; γ, between the maximum and its farther midpoint; and δ between the minimum and its farther midpoint. It can be shown (see Appendix I) that the four points on C_1 determine four chords such that pairs of opposite chords have equal products; this means that α, β, γ, and δ follow each other consecutively. The inversion with center S and radius ST carries the chart (center D, radius DS) into a complex plane of admittances

transforming point D into D' (obtained by drawing a line through T and perpendicular to DS). In the admittance plane, points D' and S have the significance of plus and minus unity, respectively, and the imaginary axis passes half way between them. The real axis passes through S and D. The admittance locus is again circle C_1 but its center D no longer is identical with the point of matched load, D'. Addition of the probe admittance Y shifts the admittance circle C_1 to a new position C_3. From C_3 we find C_2, the apparent locus of operation in the chart for the moving probe position, by applying the inversion once more, using the same center S, and unity ST as before.

These geometrical operations are the equivalent of adding the admittance Y to each point of the circle C_1 in the chart. We see now how the distorted probe pattern emerges from the construction of Fig. 4. As the ideal operating point (that is, in the absence of probe reflections) moves along circle C_1 uniformly with changing probe position, its image on circle C_2, obtained by twice inverting as described, moves at a rate which is subject to wide variations, the vector from S to the image point being proportional to the square root of the galvanometer deflection.

IV. Analysis of a Pattern

Practically, however, the problem presents itself in the reverse order, being the determination of quantities r and $Y=G+jB$ from a measured curve of the type of Fig. 1. It is not difficult to verify that the two midpoints on circle C_2 are the intersections of circle C_2 and the circle with center on line g and perpendicular to C_2, and which passes through the point S (Circle C_6, Fig. 5). Since this circle is carried into the diameter h of circle C_3 by the inversion, it becomes clear that the four characteristic probe positions are equally spaced on the admittance circle C_3, and consequently on the admittance circle C_1. The proof of this will be found in the Appendix IV. The lines g' and h' form a pair of orthogonal diameters of circle C_1, respectively parallel to lines g and h. The corresponding four points on the chart circle C_1 must, therefore, form what is known as a harmonic set, being the intersections of C_1 with two circles C_4 and C_5, both perpendicular to the circle C_1 and to each other. The circles C_4 and C_5 intersect in points D' and S; circle C_4, being related to g' by an inversion at center S, must have radius SO perpendicular to g' and thus to g. Its tangent at S is the line g which passes through the center of the circle C_5.

The analytic problem now resolves itself into the construction of Fig. 4 from the angles α and β. In order to obtain α and β, the positions of the maximum, the minimum, and one midpoint between them must be measured. The actual values of the maximum and the minimum must be known in order to calculate the value of the midpoint (equation (1)) and to determine its position. The apparent standing-wave ratio does not enter explicitly, however, in this determination of the true standing-wave ratio or the true load point in the chart. Only if it is desired to determine the probe admittance does one use the apparent standing-wave ratio explicitly.

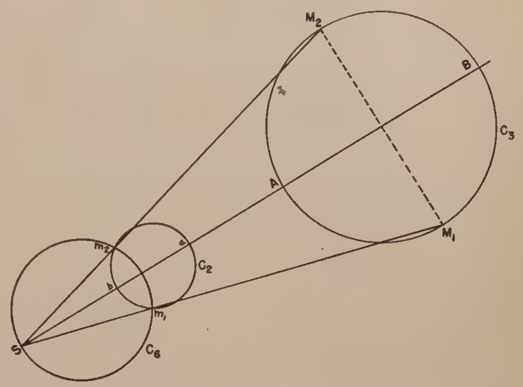

Fig. 5—Auxiliary sketch used in showing relation among maximum, minimum, and midpoint positions.

To solve the first part of the problem, a circle C_1 of arbitrary radius r is drawn to a scale to be determined later. Three points separated by angles α and β in accordance with the measured probe shifts are marked on this circle. Next, circles C_4 and C_5 are drawn through these points, both being perpendicular to the circle C_1 and to each other as shown in Fig. 2, giving S as their intersection point. The distance DS is the outer chart radius; i.e., unity, and thus by comparison gives the true value of the reflection coefficient r, and hence the true standing-wave radio $1+r/1-r$. The true chart point is obtained as the intersection of line SD and circle C_1, indicating the error in the observed minimum position. If the three characteristic points were plotted correctly, not only relative to each other but also to an assumed reference point, the chart point will also be in the correct relation to this reference point.

It is an easy matter to complete the illustration by incorporating the vector $\mathbf{Y}$. The center of circle C_3 is found by marking a distance r/r' from S along line g which is tangent to C_4 at S. The apparent reflection coefficient

$$r' = \frac{\sqrt{A} - \sqrt{B}}{\sqrt{A} + \sqrt{B}}$$

is found from the observed galvanometer readings A and B. On the other hand, the center of circle C_3 is the endpoint of vector $\mathbf{Y}$ plotted from D to a scale whose unity Y_0 is represented by the distance $SD'/2$.

These constructions are straightforward and can be carried out quickly except in cases representing extreme conditions. For instance, if maximum and minimum are approximately a quarter wave apart, circle C_4 becomes impractically large and the construction is awkward and inaccurate. For better than graphical accuracy, the reflection coefficient can be computed directly by means of the formulas

$$r = \frac{1 - \tan(\bar{\eta}/2)}{1 + \tan(\bar{\eta}/2)}$$

where

$$\sin \bar{\eta} = \frac{\sin \eta}{\sqrt{1 + \left(\frac{\sin \epsilon \cdot \sin \eta}{\cos \epsilon - \cos \eta}\right)^2}} \tag{2}$$

and

$$\eta = \frac{\alpha + \beta}{2} \qquad \epsilon = \frac{\beta - \alpha}{2}.$$

For proof, refer to Fig. 6, which is an extraction of Fig. 4, with some self-explanatory additions. From Fig. 6 it can be seen that

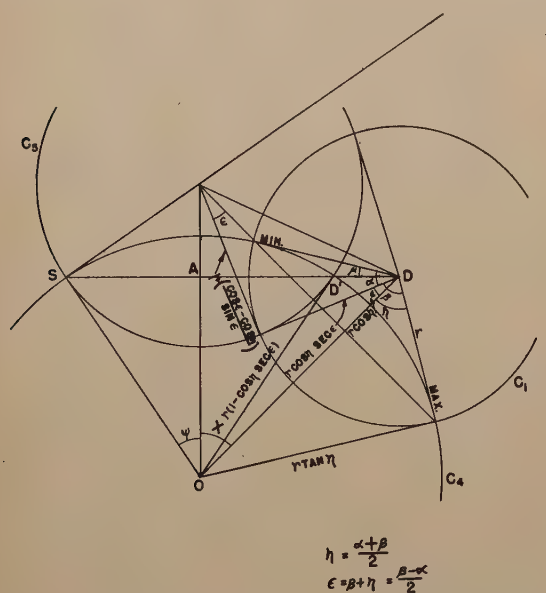

Fig. 6—Auxiliary drawing used in proof of (2) and (8).

$$\tan \psi = \frac{(\cos \epsilon - \cos \eta) \cos \eta}{\sin \epsilon \cdot \sin \eta}$$

$$\sin \psi = \frac{(\cos \epsilon - \cos \eta) \cos \eta}{\sqrt{(\cos \epsilon - \cos \eta)^2 \cos^2 \eta + \sin^2 \epsilon \cdot \sin^2 \eta}}. \tag{3}$$

$$\cos \psi = \frac{\sin \epsilon \cdot \sin \eta}{\sqrt{(\cos \epsilon - \cos \eta)^2 \cos^2 \eta + \sin^2 \epsilon \cdot \sin^2 \eta}}$$

In order to determine the unit of the admittance scale which, in Fig. 2, is represented by the length $SD'/2$, we write

$$SD' = 2r \tan \eta \cdot \sin \psi$$

$$= \frac{(\cos \epsilon - \cos \eta) \sin \eta}{\sqrt{(\cos \epsilon - \cos \eta)^2 \cos^2 \eta + \sin^2 \epsilon \cdot \sin^2 \eta}}. \tag{4}$$

Inspection of Fig. 6 will show that the following products of distances are equal:

$$(DD')(DS) = (DT)^2$$

or

$$(1)(1 - 2Y_0) = r^2. \tag{5}$$

Hence we have a quadratic equation from which the reflection coefficient may be found

$$r^2 + 2r \tan \eta \cdot \sin \psi - 1 = 0 \tag{6}$$

which together with (3) leads directly to (2).

For the probe admittance one derives (Appendix II) the formula

$$Y = \frac{2}{1 - r^2}\left[\left(\frac{r}{r'}\right) e^{j\psi} - 1\right] \tag{7}$$

where the factor $2/1 - r^2$ refers the admittance to the proper unit. The true minimum position is the point of intersection of the line SD and circle C_1 (Fig. 6). From the triangle OAD one has

$$\sin(\eta - \mu) = \frac{OA}{OD} = \frac{OA}{r \tan \eta} \cdot \frac{r \tan \eta}{OD} = \cos \psi \cdot \sin \eta \tag{8}$$

from which one computes the angle μ by which the minimum has been shifted as a consequence of probe reflections. The angle is counted positive toward the nearer of the two midpoints.

In concluding the general discussion, it should be mentioned that the chart point can also be determined in terms of the quantities α and δ (Fig. 4). The introduction of the angle δ instead of β requires little labor, since (2) remains correct when the letter β in the definitions of ϵ and η is replaced by δ and the letter ψ in (3) by ψ'. Equation (8) then remains valid with the same changes. This form will be of use later when we take up the case of high standing-wave ratios.

V. Experimental Verification

All measurements were performed with a standing-wave detector of standard design using a rectangular wave guide and operating at wavelengths of a few centimeters. The detector was coupled to the electric field by means of a probe wire 0.010 inch in diameter which protruded into the wave guide through a 3/32-inch slot. The arrangement of the apparatus is shown schematically in Fig. 7. At the right in the illustration is the

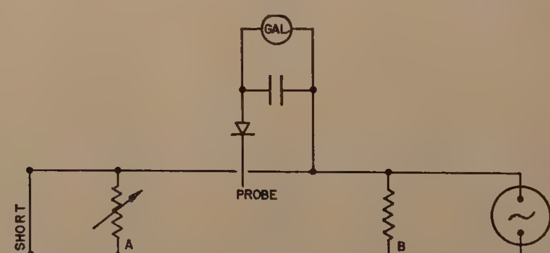

Fig. 7—Schematic arrangement showing wave detector and variable load.

generator which is buffered by the fixed attenuator B. At the left in the illustration is shown the load which consists of a variable attenuator A backed by a short circuit. The attenuator B was matched to the characteristic impedance of the transmission line, and was of such a value that the generator was always working into a nearly matched load regardless of the position or the setting of the attenuator A. The generator, a reflex klystron, was operated from a well-regulated power supply.

The standing-wave detector was completely overhauled prior to the measurements, and with a matched

load the probe response was constant within one per cent all along the length of the slot. A low-resistance, critically damped galvanometer served as the indicator of probe response. All attenuators were calibrated and the square-law response of the probe crystal was carefully checked.

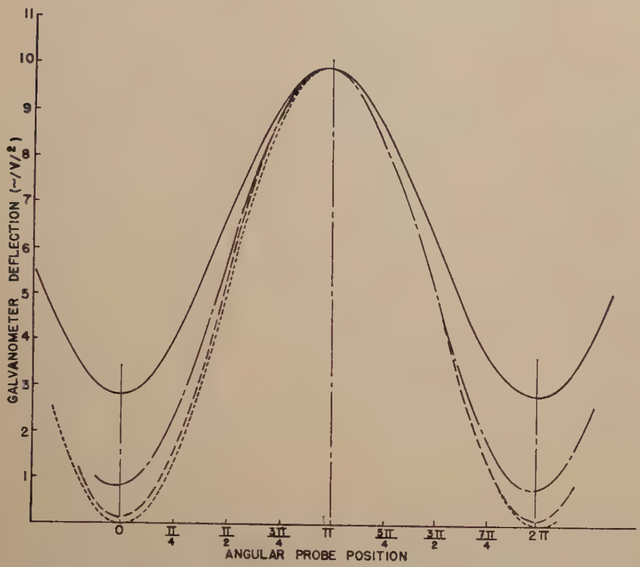

Fig. 8—Standing-wave patterns as a function of load. Curves experimental. Constant probe penetration 25 per cent.

0.31
0.56 Reflection coefficient
0.82
1.00

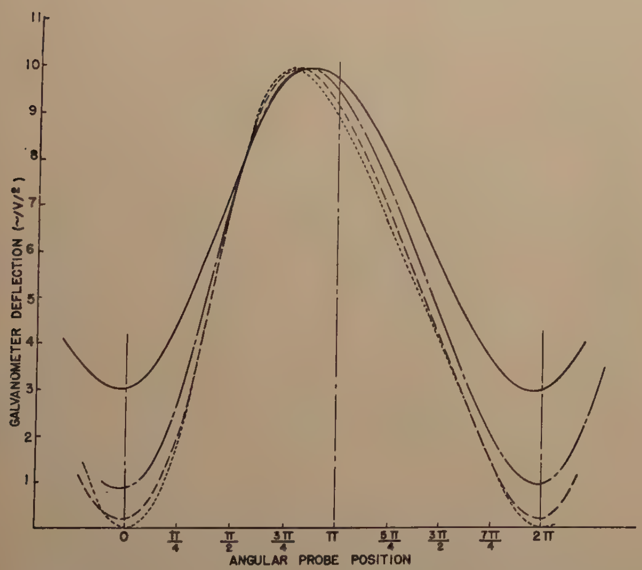

Fig. 9—Standing-wave patterns as a function of load. Curves experimental. Constant probe penetration 50 per cent.

0.31
0.56 Reflection coefficient
0.82
1.00

As a sample of the computations of Section III, the true reflection coefficient r and the probe admittance $Y=G-jB$ have been computed for a fixed load and a number of probe penetrations, and tabulated in Table II. The load was the same as that used in Fig. 4. Probe penetration is quoted in per cent of guide height. The tabulated results bear out the contention that the probe acts as a shunt admittance so long as the penetration is not more than 65 per cent, and the theory presented here should thus be quite satisfactory in all practical standing-wave measurements.

In Figs. 8, 9, 10, 11, and 12 is shown the variation of probe patterns with load, the probe penetration remaining constant. For penetrations up to 25 per cent, the shift of maximum and minimum is not noticeable (Fig. 8), but this is no longer true for penetrations between 25 and 50 per cent (Fig. 9). For penetrations exceeding 50 per cent (Figs. 10, 11, and 12) the pattern departs noticeably from a sine curve beyond the shifts of maximum and minimum.

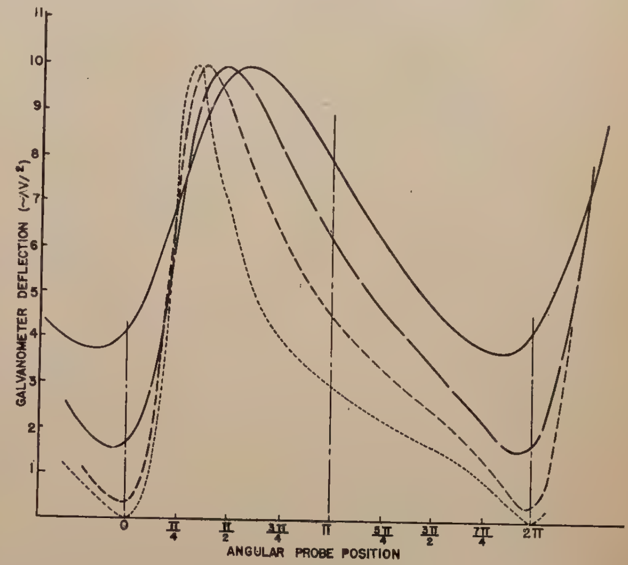

Fig. 10—Standing-wave patterns as a function of load. Curves experimental. Constant probe penetration 75 per cent.

0.31
0.56 Reflection coefficient
0.82
1.00

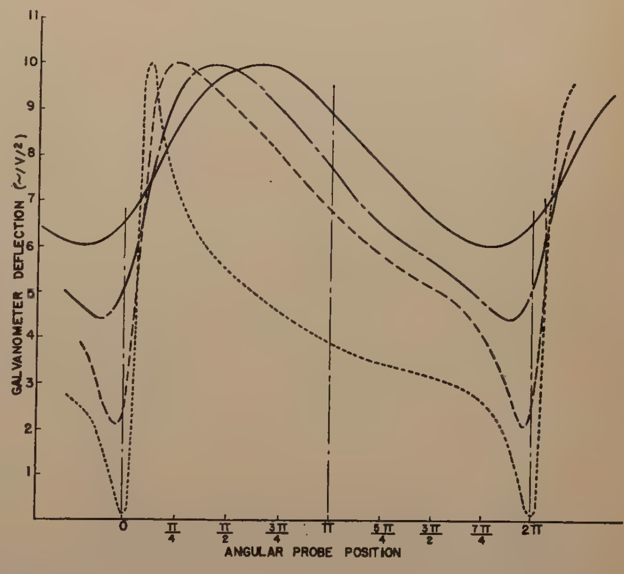

Fig. 11—Standing-wave patterns as a function of load. Curves experimental. Constant probe penetration 90 per cent.

0.31
0.56 Reflection coefficient
0.82
1.00

The effect of increasing probe penetration on the pattern for a given load (Figs. 13, 14, 15, and 16) is generally to reduce the apparent standing-wave ratio. In these, as in the preceding illustrations, the appearance of more than one maximum per half wave for extremely deep penetrations is clearly unaccounted for by the shunt-admittance theory and thus affords a sufficient, though not a necessary, criterion that the formulas of Section III are no longer applicable.

Fig. 17 shows four curves calculated for widely different probe admittances, together with experimental points found with probe penetrations such as to approximate the calculated curves most closely. The deviations are not very striking even for the larger probe penetrations of above 75 per cent, yet remind us that our assumptions should not be carried to probe penetrations beyond 65 per cent.

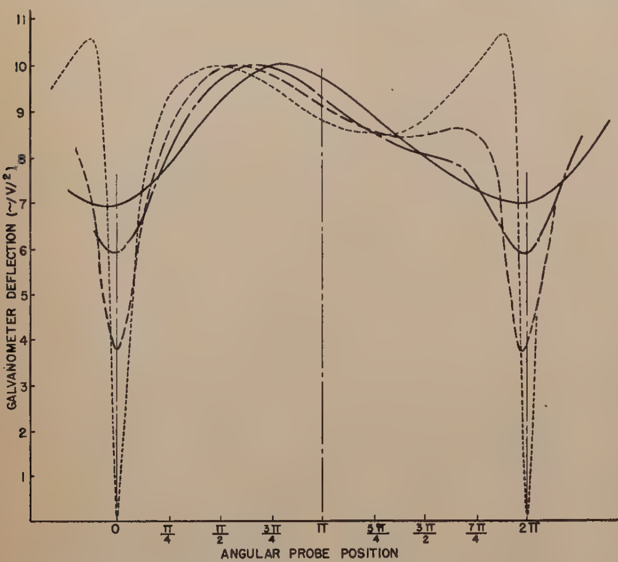

Fig. 12—Standing-wave pattern as a function of load. Curves experimental. Constant probe penetration 97 per cent.
——————— 0.31
—— – —— – —— 0.56 Reflection coefficient
– – – – – – – – – 0.82
----------------------- 1.00

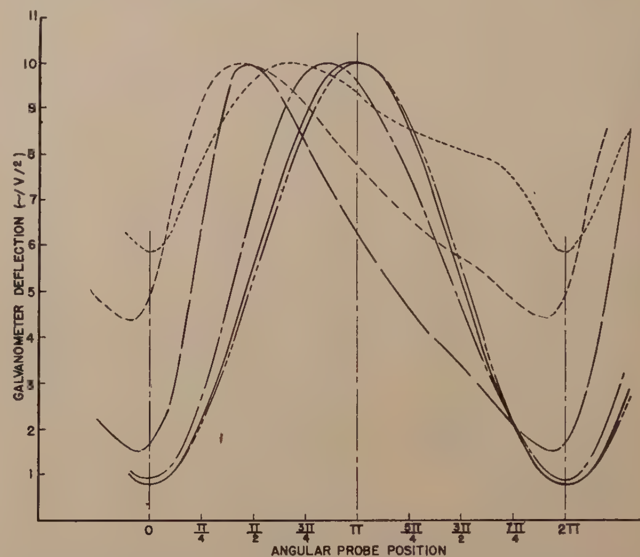

Fig. 14—Standing-wave patterns as a function of probe penetration Curves experimental. Constant load reflection coefficient 0.56.
——---—— 10 per cent
——————— 25 per cent
—— – —— – —— 50 per cent
—— —— —— 75 per cent
– – – – – – – – – 90 per cent
----------------------- 97 per cent

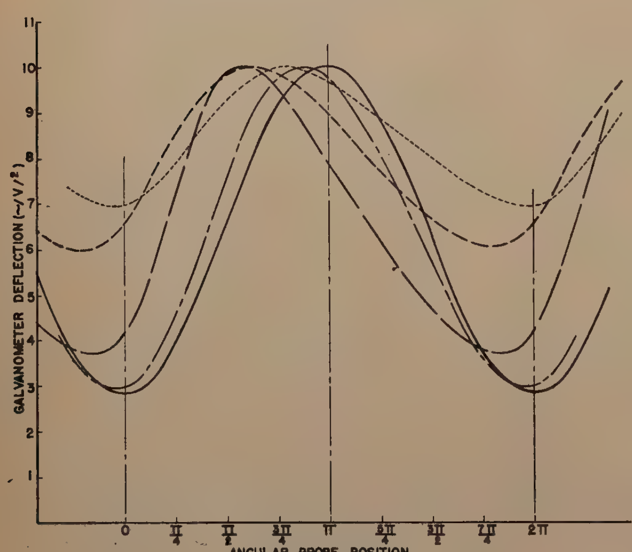

Fig. 13—Standing-wave patterns as a function of probe penetration. Curves experimental. Constant load reflection coefficient 0.31.
——————— 25 per cent
—— – —— – —— 50 per cent
—— —— —— 75 per cent
– – – – – – – – – 90 per cent
----------------------- 97 per cent

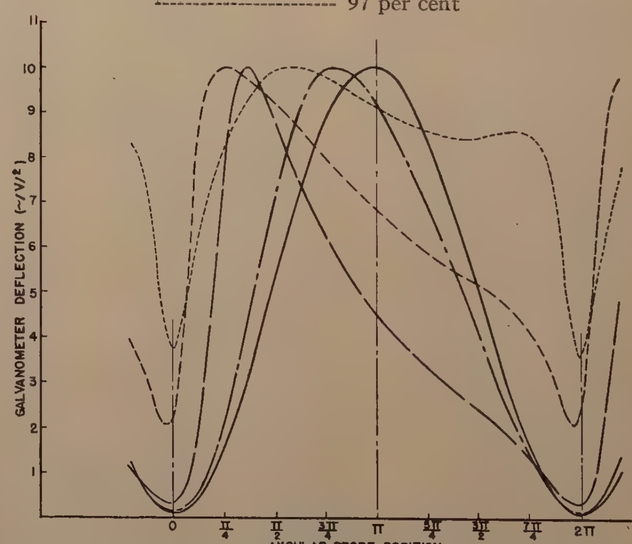

Fig. 15—Standing-wave patterns as a function of probe penetration. Curves experimental. Constant load reflection coefficient 0.82.
——————— 25 per cent
—— – —— – —— 50 per cent
—— —— —— 75 per cent
– – – – – – – – – 90 per cent
----------------------- 97 per cent

Table III is a compilation of all measurements. In each section, representing a constant load, the computed reflection coefficients are correct for penetrations below 65 per cent and approximately correct for 75 per cent.

The probe admittance increases rapidly with probe penetration for a given load and, contrary to what might be expected, varies also with the load when the

penetration is kept constant. This is not quite so surprising considering that the probe admittance, particularly in its conductive component, is a sensitive function of the tuning of the standing-wave detector. An attempt was made to readjust the tuning for each probe penetration so as to obtain maximum response, yet the probe admittance still showed variations independent of the penetration.

As an additional and more direct check on the computed values, the probe admittances were subjected to measurement by means of the experimental setup shown schematically in Fig. 18. The only changes compared with Fig. 7 are the added standing-wave detector between the first standing-wave detector and the buffer attenuator, as well as a carefully matched load substituted for the variable load of Fig. 7. The admittance

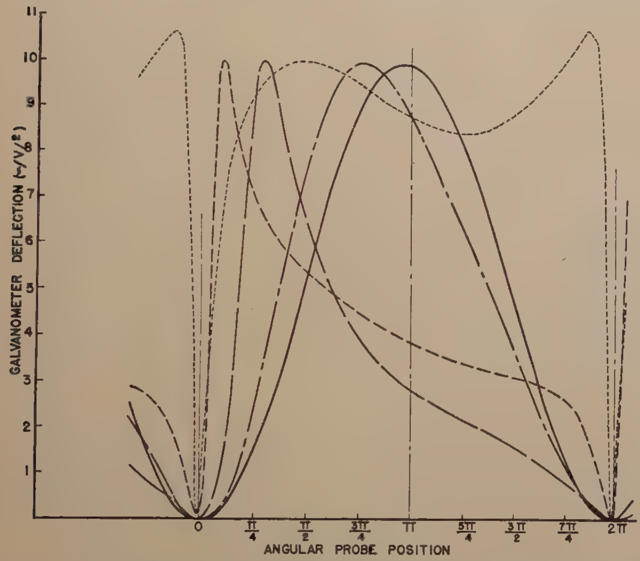

Fig. 16—Standing-wave patterns as a function of probe penetration. Curves experimental. Constant load reflection coefficient 1.00.

———————— 25 per cent
— — — — — 50 per cent
——— ——— 75 per cent
- - - - - - - - 90 per cent
------------------- 97 per cent

values measured with the second detector and the computed values are plotted in the Smith chart (Fig. 19), showing good agreement for a number of probe penetrations.

VI. Special Case of High Standing-Wave Ratios

For very high standing-wave ratios, the situation requires a separate discussion, since the formulas of Section III while, of course, still valid, are no longer practical. In the first place, the reading of maximum and minimum at the same power level is impractical and it is expedient to use the two midpoints and the minimum instead. The midpoints may now be taken as the probe positions with twice minimum response, separated from the minimum position by angles α and δ. We must, therefore, avail ourselves of the modified form of (2) as was mentioned at the end of Section IV.

In the second place, with increasing standing-wave ratios, the two midpoints move closer and closer toward the minimum position and the two spacings become equal in the limit. Thus, the value of ϵ is small and of higher order than η. This works toward a simplification of the formulas, the net result being the same as if reflections were ineffective. This is true, notwithstanding

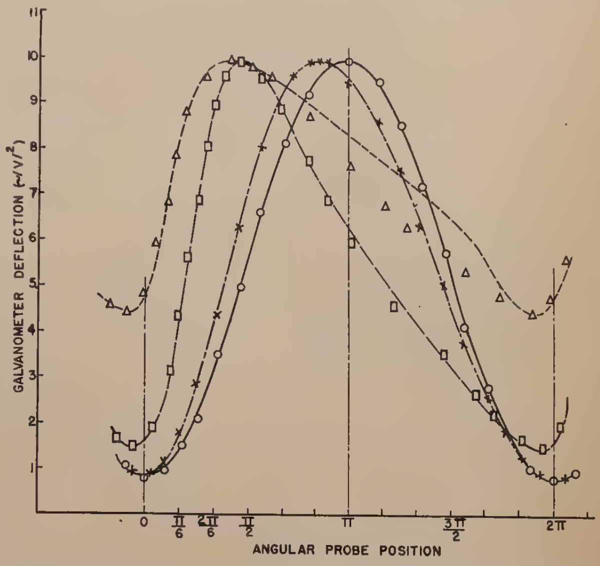

Fig. 17—Comparison of experimental and calculated patterns. Points experimental, curves calculated. Constant load reflection coefficient 0.56.

—O—O—O—O 25 per cent
—X—X—X—X 50 per cent
—□—□—□—□ 75 per cent
---△---△---△---△ 90 per cent

the fact that the true and the apparent standing-wave ratios differ considerably. The situation is best understood by considering the effect of increasing probe penetration. Starting with an ideal probe one would obtain a strictly sinusoidal pattern. As the probe admittance becomes considerable, the pattern is distorted by a shift and lowering of the maximum, and yet the distorted patterns still pass very nearly through the same

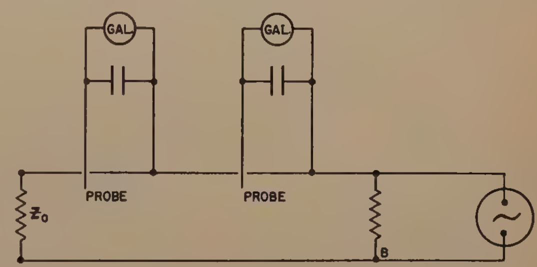

Fig. 18—Schematic diagram showing arrangement for direct measurement of probe admittance.

minimum and midpoints. For power standing-wave ratio of 400, the correction term becomes much less than 1 per cent.

A series expansion of the modified equation (2) in powers of η (see Appendix 5) gives

$$r = \frac{1 - \tan \eta/2}{1 + \tan \eta/2} + \frac{\epsilon^2}{2} \frac{1 + \cos \eta}{1 - \cos \eta} \frac{\tan \eta}{(1 + \sin \eta)}$$

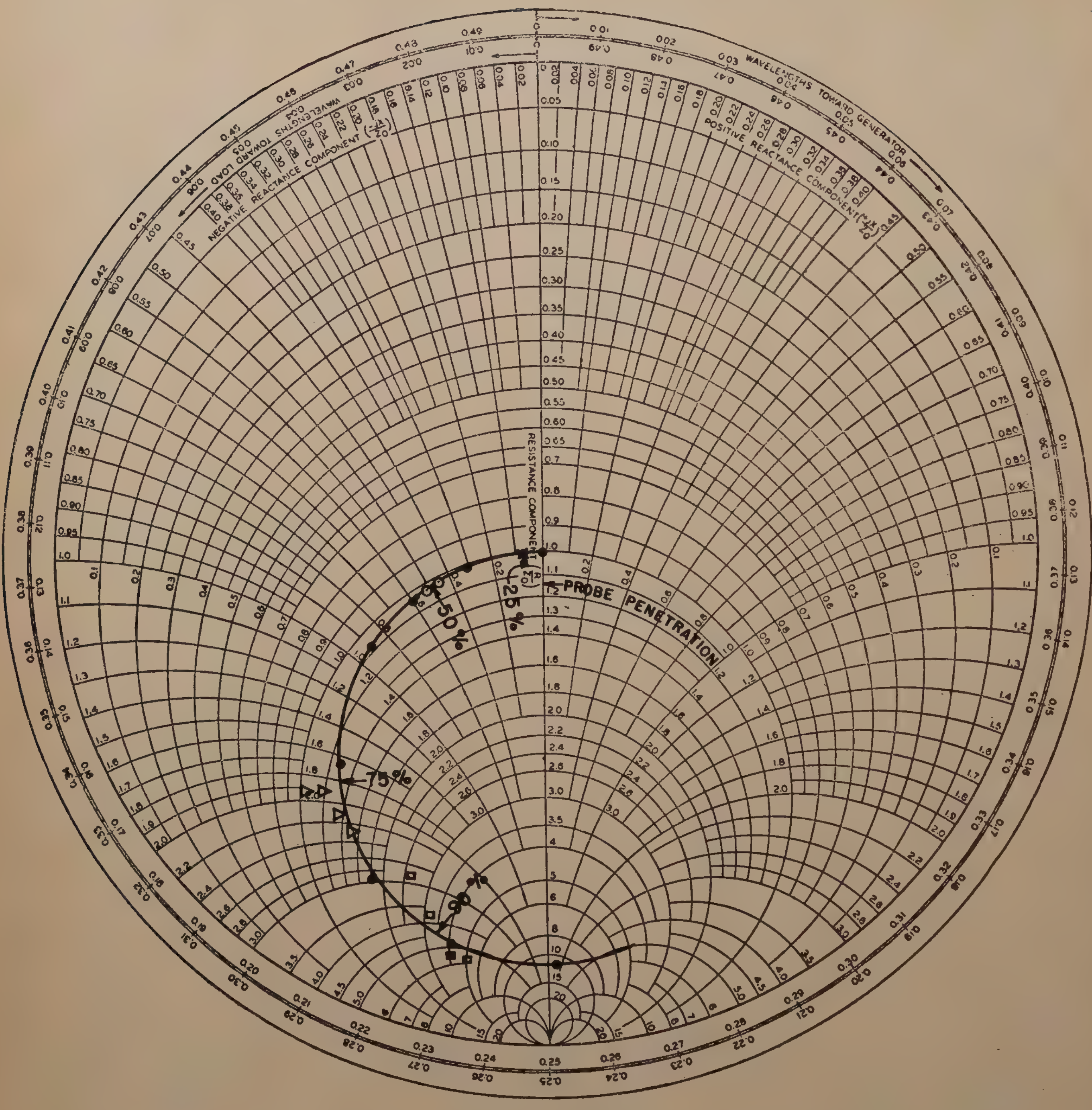

Fig. 19—Comparison of experimental and calculated probe admittances.

● = probe admittance measured
× = probe admittance calculated 25 per cent
○ = probe admittance calculated 50 per cent
△ = probe admittance calculated 75 per cent
□ = probe admittance calculated 90 per cent

and since η itself is small

$$1 - r = \frac{2}{1 + \cot \eta/2} - \frac{2\epsilon^2}{\eta}.$$

A direct determination of ϵ would be inaccurate owing to the smallness of α and δ plus the uncertainty attending the exact minimum position. The correction term can be found with much greater accuracy by means of the maximum position, which shows very considerable shifts even for slight asymmetry of the minimum versus midpoints. By virtue of Appendix I we can write

$$\frac{\alpha}{\delta} \cong \frac{\sin \alpha/2}{\sin \delta/2} = \frac{\sin \frac{360 - \gamma - \alpha - \delta}{2}}{\sin \gamma/2}$$

$$= \cos \eta + \cot (\gamma/2) \sin \eta$$

and

$$\epsilon/\eta = \eta/2 \cot \gamma/2 + \cdots.$$

Finally

$$1 - r = \frac{2}{1 + \cot (\eta/2)} - \eta^3/2 \cot^2 \gamma/2$$

$$\cong \eta - \eta^2/2 - \eta^3/2 \left[\cot^2 \gamma/2 - \frac{5}{12} \right]. \quad (9)$$

Inspection of Fig. 16 shows that the angle γ stays well above 45 degrees for probe penetrations up to 75 per cent, so that the third-order correction term at its worst will amount to less than 3 per cent of the total $(1-r)$ if $r \geqq 0.90$.

Appendix I

Relation Between Four Harmonic Points

Four points (A, B, C, and D) in the complex plane which can by a linear transformation be transformed into four equidistant points on a circle are a harmonic set which means that the anharmonic cross ratio λ $(A, C; B, D)$ of their complex values is

$$\lambda(A, C; B, D) = \frac{(A - B)(C - D)}{(A - D)(B - C)} = -1.$$

Each of the four brackets represents a cord of the circle on which the four points lie, subtending the angles α, β, γ, and δ respectively. It is then easily seen that

$$\sin \alpha/2 \sin \gamma/2 = \sin \beta/2 \sin \delta/2$$

and this is the relation sought.

Appendix II

Computation of the Complex Probe Admittance

In the triangle formed by vector Y and line SD (Fig. 4) the third side has the length r/r' and is rotated through ψ. Since the vector SD is the unit vector, we can write down the vector relation

$$1 + Y = r/r' e^{i\psi}.$$

To write down the expression for Y, it is necessary to refer it first to the proper unit Y_0 which, in the illustrations, is represented by distance

$$\frac{SD'}{2} = \frac{1 - r^2}{2}.$$

(Note that line TD' in Fig. 4 is at right angles to SD.) Hence

$$G = 2/1 - r^2 \left[\frac{r}{r'} \cos \psi - 1 \right]$$

$$B = 2/1 - r^2 \left[\frac{r}{r'} \sin \psi \right].$$

Appendix III.

Proof of Fig. 2(b)

In Fig. 4 the circle C_2 was obtained by inverting the circle C_1 in the point S, using ST as a radius, adding the vector Y to C_1 in the rectangular admittance plane giving C_3, and then again inverting C_3 at S, using radius ST. Fig. 2(b) represents exactly the same process with the exception that the vector Y is subtracted from S rather than added to C_1. Here C_1 is inverted in S, using radius ST (carrying P into V), and Y subtracted from S. This latter step leaves circle C_1 in the same relation to point O as circle C_3 is to the point S in Fig. 4. All that remains now is the inversion of C_1 in the point O to get a circle equivalent to C_2 of Fig. 4. If ST were used as radius, we would obtain a circle on the same scale as C_2 but to simplify the construction we will change scales and use as inversion radius a tangent to C_1 through point O. This inversion carries the point V into the point Q.

The result is, therefore, that with a true load point P, the voltage as measured with a probe of admittance X, will be proportional to the vector OQ rather than to the vector SP. We may thus compute probe patterns, if we are given the probe admittance Y and the true load-reflection coefficient r.

Appendix IV

Equal Spacing of the Four Characteristic Probe Positions on Circle C_3

Referring to Fig. 5, which is an excerpt of Fig. 4, we remember that circle C_2 was obtained from circle C_3 by an inversion in the point S, using radius ST. This carries points A, B, M_1, and M_2 into points a, b, m_1, and m_2 respectively, so that the following relation holds:

$$\overline{SA} \cdot \overline{Sa} = \overline{SB} \cdot \overline{Sb} = \overline{SM_1} \cdot \overline{Sm_1} = \overline{SM_2} \cdot \overline{Sm_2} = \overline{ST}^2. \quad (10)$$

Now the maximum of the standing-wave pattern is proportional to $\overline{Sa}^2$, the minimum to $\overline{Sb}^2$, and the midpoint (1) to

$$\frac{2\,\overline{Sa}^2 \cdot \overline{Sb}^2}{\overline{Sa}^2 + \overline{Sb}^2} = \overline{Sm_1}^2 = \overline{Sm_2}^2. \quad (1')$$

In Fig. 5, if M_1, B, M_2, and A are equally spaced on circle C_3, we have by geometry

$$\overline{SM_1}^2 = \frac{\overline{SA} + \overline{SB}^2}{2} + \frac{\overline{SB} - \overline{SA}^2}{2} = \overline{SM_2}^2$$

TABLE II

REFLECTION COEFFICIENT =0.56
VARIOUS PROBE PENETRATIONS

Probe Penetration	10 per cent	25 per cent	50 per cent	75 per cent	90 per cent
Maximum	10.00	10.00	10.00	10.00	10.00
Position of Maximum	0.394	0.382	0.336	0.210	0.192
Minimum	0.80	0.80	0.87	1.52	4.40
Position of Minima	0.038	0.035	0.033	0.013	0.000
	0.752	0.756	0.748	0.728	0.712
Midpoint	1.48	1.48	1.60	2.64	6.11
Position of Midpoint	0.100	0.098	0.091	0.072	0.059
	0.690	0.688	0.681	0.626	0.524
α (in degrees)	31	30	30	30	30
β (in degrees)	149	144	123	72	67
γ (in degrees)	149	154	174	208	168
δ (in degrees)	31	32	33	50	95
η (in degrees)	90	87	76.5	51	48.5
ϵ (in degrees)	59	57	46.5	21	18.5
r (calculated)	0.56	0.56	0.56	0.55	0.52
r' (apparent r)	0.56	0.56	0.54	0.46	0.23
r/r'	1.00	1.00	1.03	1.20	2.26
$\sin\psi$	0.001	0.026	0.147	0.600	0.630
$\cos\psi$	1.00	1.00	0.99	0.87	0.79
$2/1-r^2$	2.91	2.91	2.91	2.82	2.73
B (probe)	0.003	0.08	0.45	1.98	3.88
G (probe)	0	0	0.06	0.12	2.14

TABLE III

VARIOUS PROBE PENETRATIONS

(a) Reflection Coefficient =0.31

Probe Penetration	25 per cent	50 per cent	75 per cent	90 per cent
Maximum	10.00	10.00	10.00	10.00
Minimum	2.81	2.94	3.70	6.00
α (in degrees)	54	53	51	57
β (in degrees)	124	112	82	89
γ (in degrees)	126	135	144	122
δ (in degrees)	56	60	83	92
r (calculated)	0.31	0.31	0.30	0.25
r' (apparent)	0.31	0.30	0.24	0.12
G (probe)	0.00	0.04	0.09	1.49
B (probe)	0.04	0.50	1.92	2.68

(b) Reflection Coefficient =0.56

Probe Penetration	10 per cent	25 per cent	50 per cent	75 per cent	90 per cent
Maximum	10.00	10.00	10.00	10.00	10.00
Minimum	0.80	0.80	0.87	1.52	4.40
α (in degrees)	31	30	30	30	30
β (in degrees)	149	144	123	72	67
γ (in degrees)	149	154	174	208	168
δ (in degrees)	31	32	33	50	95
r (calculated)	0.56	0.56	0.56	0.55	0.52
r' (apparent)	0.56	0.56	0.54	0.46	0.23
G (probe)	0.00	0.00	0.06	0.12	2.14
B (probe)	0.00	0.08	0.45	1.98	3.88

(c) Reflection Coefficient =0.82

Probe Penetration	25 per cent	50 per cent	75 per cent	90 per cent
Maximum	10.00	10.00	10.00	10.00
Minimum	0.11	0.16	0.30	2.05
α (in degrees)	11	11	11	12
β (in degrees)	166	131	57	37
γ (in degrees)	172	205	276	289
δ (in degrees)	11	13	16	22
r (calculated)	0.82	0.82	0.81	0.79
r' (apparent)	0.82	0.78	0.70	0.38
G (probe)	0.00	0.06	0.34	4.00
B (probe)	0.04	0.44	2.11	6.40

(d) Reflection Coefficient =1.00

Probe Penetrations	25 per cent	50 per cent	75 per cent	90 per cent
Maximum	10.00	10.00	10.00	10.00
Minimum	0.00	0.00	0.001	0.02
α (in degrees)	0	0	0	1
β (in degrees)	176	140	56	20
γ (in degrees)	184	220	304	338
δ (in degrees)	0	0	0	1
r (calculated)	1.00	1.00	1.00	0.99
r' (apparent)	1.00	1.00	0.98	0.91
G (probe)	0.00	0.06	0.60	3.00
B (probe)	0.04	0.40	2.30	5.30

or

$$\frac{1}{\overline{SM_1}^2} = \frac{2}{\overline{SA}^2 + \overline{SB}^2} = \frac{1}{\overline{SM_2}^2}.$$

And substituting from (10) we have

$$\overline{Sm_1}^2 = \frac{2\,\overline{Sa}^2 \cdot \overline{Sb}^2}{\overline{Sa}^2 + \overline{Sb}^2} = \overline{Sm_2}^2.$$

Which is (1′), the definition of our midpoints, thus proving that the inversion of the midpoints and the maximum and minimum give four equally spaced points on circle C_3.

APPENDIX V

SERIES EXPANSION OF (2) FOR HIGH STANDING-WAVE RATIOS

From the definition of the equation for $\bar{\eta}$ we have

$$\tan\frac{\bar{\eta}}{2} \equiv \frac{1-\cos\bar{\eta}}{\sin\bar{\eta}} = \frac{\sqrt{1+\left(\dfrac{\sin\epsilon\cdot\sin\eta}{\cos\epsilon-\cos\eta}\right)^2} - \sqrt{\cos^2\eta+\left(\dfrac{\sin\epsilon\cdot\sin\eta}{\cos\epsilon-\cos\eta}\right)^2}}{\sin\eta}$$

$$= \frac{1-\cos\eta}{\sin\eta} + \frac{\epsilon^2}{2}\,\frac{\sin\eta}{(1-\cos\eta)^2}\left(1-\frac{1}{\cos\eta}\right) = \tan\frac{\eta}{2} - \frac{\epsilon^2}{2}\,\frac{\tan\eta}{(1-\cos\eta)}.$$

Inserting this into (2), we have

$$r = \frac{1-\tan\eta/2}{1+\tan\eta/2} + \epsilon^2\frac{\tan\eta}{(1-\cos\eta)(1+\tan\eta/2)^2} = \frac{1-\tan\eta/2}{1+\tan\eta/2} + \frac{\epsilon^2}{2}\,\frac{1+\cos\eta}{1-\cos\eta}\,\frac{\tan\eta}{1+\sin\eta}.$$

Contributors

William Altar

William Altar obtained the Ph.D. degree in physics and mathematics at the University of Vienna in 1923, and later received training in electrical engineering from the Technical University in Vienna. After several years as an industrial engineer, Dr. Altar engaged in research work on wave propagation at King's College, London.

He came to the United States in 1929 as an assistant professor of physics at the Pennsylvania State College, a position which he held until 1935, when he was awarded a two-year Fellowship in chemical physics at Frick Chemical Laboratory, Princeton University. Following this study, Dr. Altar spent two years in Istanbul, Turkey, where he taught electrical engineering at Roberts College.

From 1940 to 1942 he taught electrical engineering at the Case School of Applied Science, and in 1942 became associated with the Westinghouse Research Laboratories, as an engineer engaged in work on microwaves. He is a member of the American Institute of Electrical Engineering.

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Sidney T. Fisher

Sidney T. Fisher (M'42–SM'43) was born in Edmonton, Alberta, Canada, in 1908. He received the B.A.Sc. degree in electrical engineering from the University of Toronto in 1930.

During 1928 and 1929, and from 1930 to 1943, Mr. Fisher was with the special products division of the Northern Electric Company, Ltd., at Montreal, where he was engaged successively in power-tube engineering, field-engineering, and installation of public-address systems and broadcast stations, and development and design of radio- and audio-frequency equipment of all types, becoming sales engineer and development engineer in 1941. During this period, he was responsible for the engineering of the radio installations for Canadian-built Lancaster and Mosquito bombers. Mr. Fisher resigned from the Northern Electric Company in 1943 to organize F. T. Fisher's Sons, Ltd., consulting engineers, and has subsequently carried out engineering projects for Defence Communications, Ltd., the Royal Canadian Air Force, and other service and government groups. In 1943 he became vice-president of Rogers Electronic Tubes, Ltd., of Toronto. Since 1941, he has held a commission as Flying Officer in the Special Reserve of Officers (technical) of the R.C.A.F., and in this capacity has worked with the R.C.A.F. on air-force signals problems.

Mr. Fisher is a member of the Engineering Institute of Canada, the American Institute of Electrical Engineers, and the Accoustical Society of America.

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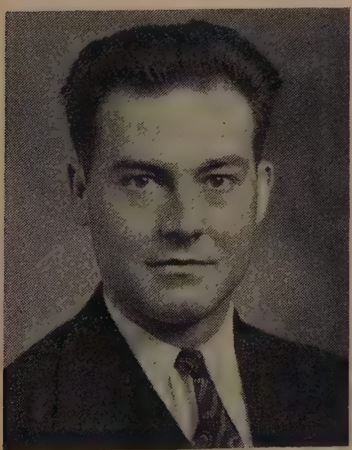

L. P. Hunter

L. P. Hunter received the B.S. degree in physics from the Massachusetts Institute of Technology, the B.A. degree from the College of Wooster, and the D.Sc. degree from Carnegie Institute of Technology, in 1942.

In 1942, Mr. Hunter joined the Westinghouse Electric Corporation, engaging in work on the physics of solids. His first assignment was a study of the behavior of the elastic constants of copper-aluminum alloys, and on crystalline rock salt near the melting temperature. Since that time he has been engaged in microwave development, and has done research on the Manhattan project. He is now associated with X-ray research.

Dr. Hunter is a member of the American Physical Society.

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Robert A. Kirkman

Robert A. Kirkman was born in 1915, at Portland, Oregon. He was graduated from the RCA Institutes in 1937, and has been active in amateur radio since 1931. In 1935 and 1936, Mr. Kirkman was associated with the Teleradio Engineering Corporation, in New York City, and in 1938 was employed in the New York office of the General Electric Company. He left this position to work with the City of New York, first on the radio network of the New York City Fire Department, and later with the Municipal Broadcasting System.

Since 1941 he has been connected with the Signal Corps Engineering Laboratories, at Fort Monmouth, New Jersey. He served in the radio direction-finding branch as project engineer, section engineer, and in de-

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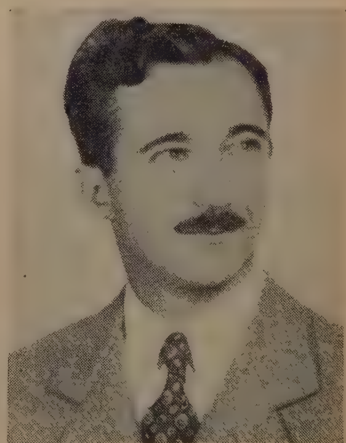

Morris Kline

Contributors

Everard M. Williams

velopment of types of radio and radar direction finders and electronic meteorological equipment.

He was elected a member of the Board of Directors of the American Radio Relay League in 1940, 1942, and 1944.

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Morris Kline was born at Brooklyn, New York, in 1908. He received the B.Sc. degree in mathematics from New York University in 1930, and the Ph.D. degree in mathematics in 1936 from the same institution. From 1930 to 1936, he was an instructor in mathematics at New York University, and served as research assistant in the Institute for Advanced Study of Princeton University from 1936 to 1938, returning as an instructor in mathematics to New York University from 1938 to 1942. He lectured in the graduate division of the Hunter College from 1939 to 1940.

Dr. Kline was appointed a radio engineer in the Signal Corps Engineering Laboratories, and from 1942 to 1945, served as project engineer in charge of the development of meteorological radio direction finders. He is at present assistant professor of mathematics at New York University, and consultant on mathematics to the Reeves Sound Laboratories. He is co-author of a college textbook on mathematics.

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Everard M. Williams (S'36–A'41–SM'44) was born at New Haven, Connecticut, on February 2, 1915. He received the B.E. degree in 1936 and the Ph.D. degree in 1939 from Yale University. During the summer of 1937 he was employed by the General Electric Company, and during the academic year 1938 and 1939 he was the recipient of a Charles A. Coffin Fellowship from this company. From 1939 to 1942, he was an instructor in electrical engineering at the Pennsylvania State College. Since 1942 he has been development branch engineer in what is now the special projects laboratory, radio and radar subdivision, Air Technical Service Command (Army Air Forces), Wright Field, Dayton, Ohio. He is a member of Tau Beta Pi, Sigma XI, and the Society for the Promotion of Engineering Education.

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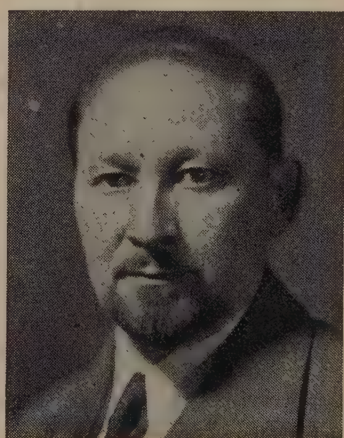
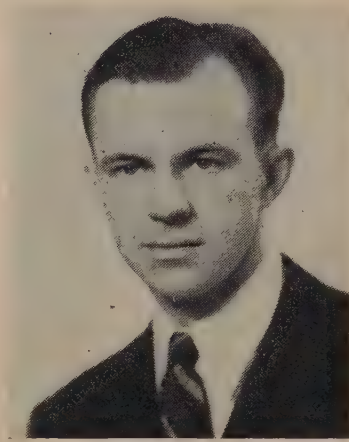


S. A. Schelkunoff



F. B. Marshall

F. B. Marshall was graduated from the Southern Methodist University in 1933, and subsequently received the Ph.D. degree in physics from the University of Chicago. In 1940, he became curator of exhibits at Buhl Planetarium, in Pittsburgh, Pennsylvania, and also taught physics and advanced X-ray theory at the University of Pittsburgh.

Dr. Marshall became a research engineer in electronics for the Westinghouse Electric Corporation in 1942, engaged in original work on microwaves, and is now associated with work on X-ray. He has also been actively interested in progressive education.

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S. A. Schelkunoff (A'40–F'44) received the B.A. and M.A. degrees in mathematics from the State College of Washington in 1923, and the Ph.D. degree in mathematics from Columbia University in 1928. He was in the engineering department of the Western Electric Company from 1923 to 1925; the Bell Telephone Laboratories from 1925 to 1926; the department of mathematics of the State College of Washington, 1926 to 1929; and Bell Telephone Laboratories, 1929 to date. Dr. Schelkunoff has been engaged in mathematical research, especially in the field of electromagnetic theory.

An Explanatory Statement

Concerning

THE PROCEEDINGS OF THE I.R.E.

and

WAVES AND ELECTRONS

In placing before the membership of The Institute of Radio Engineers the first issue of a new, dual publication of the Institute containing *two* technical journals, rather than one as heretofore, it is believed appropriate to explain the purposes, policies, and procedures adopted for the two journals to the extent that these have been crystallized up to the present and, upon recommendation, approved by the Board of Directors.

It has long been evident that the material appearing in the PROCEEDINGS OF THE I.R.E. covered a wide range of subject matter and treatment. For convenience of reference, ease of binding, comfort in reading, adaptation to the needs of the Institute membership, and adjustment to the trends of the field of the Institute, it has seemed desirable to divide the technical material available for publication into two major groups, and to include each of these groups in a separate and conveniently usable periodical. The provision of two such diversified journals will permit the more flexible expansion or modification of each of them, and thus will doubtless enhance their utility to the membership of the Institute.

Division of Material

The division of material between the PROCEEDINGS OF THE I.R.E., on one hand, and WAVES AND ELECTRONS, on the other hand, can be made reasonably clear in its present status through the following discussion:

Appearing in the PROCEEDINGS OF THE I.R.E. generally will be papers dealing with advanced research topics; the more advanced types of equipment and method development; mathematico-physical analyses of engineering problems; relatively advanced or abstract studies of scientific or technical matters; Standards Reports formally adopted by The Institute of Radio Engineers; adopted reports of the Technical Committees of the Institute; and other matters which are likely to be found of fundamental or basic nature.

In the pages of WAVES AND ELECTRONS, there will generally be included, according to present plans, papers dealing with current engineering developments of equipment or methods; tutorial papers covering specific aspects of radio-and-electronic engineering in an authoritative and clear form and readily usable for "refresher" purposes by our engineering readers and Student members; historical papers tracing the development of concepts or aspects of apparatus or methods

now in common use and setting forth the trends of the radio-and-electronic art, papers dealing with engineering welfare matters, including the problems, viewpoints, and aspirations of our engineering members; reports of standards adopted by organizations other than the Institute, together with the membership of the committees of such organizations responsible for the standards in question; information concerning the activities and accomplishments of the members of the Institute; intramural news items dealing with the activities of the Institute, its Sections, its Committees, its Officers, and its Staff, together with the elements of its collaboration with other learned societies; book reviews of current interest; and bibliographical abstracts in such forms as may become available.

It is to be expected that the list of items included in both the PROCEEDINGS OF THE I.R.E. and WAVES AND ELECTRONS will be expanded or modified as time goes on and experience develops methods whereby these periodicals may be made of greater service to the membership. However, the foregoing discussion indicates the nature of present plans to an extent which will enable our readers to form a judgment relative to their utility.

It should be here emphasized that high technical standards and careful editorial-review procedures will be maintained in both journals, and that papers in each of them are regarded as on an equal basis professionally. The Editorial Department of the Institute has encountered numerous and difficult problems in the preparation and issuance of this first issue of our dual technical journals. For the present, these journals will be published under a single cover, although it will be readily possible for the readers to separate these either immediately upon receipt, or at the end of each year for binding into appropriate volumes. Each journal, it will be found, has its own contents page and numbered page sequence.

Comment of Readers Invited

The future usefulness and success of both journals will be dependent in considerable measure upon the extent to which the readers of these journals convey their views to the Institute. The Editorial Department will, accordingly, be grateful for any comments which our readers may make concerning any aspect of these journals and for any suggestions as to modifications, additions, omissions, or changes in policy. The Institute of Radio Engineers and its Editorial Department exist to serve the membership, and they will best accomplish this aim if the membership is thoroughly articulate. Each reader addressing the Editorial Department may be assured that his views will be carefully considered and analyzed and, so far as is consistent with basic policy, the professional standards of the Institute, operating practicability, and a reasonable support for these viewpoints, will be put into practice. Such expressions of opinion should be addressed to the Editorial Department of the Institute at 26 West 58th Street, New York 19, New York. In the meantime, the friendly support of the membership of the Institute is earnestly solicited for the two journals which are submitted to them at this time.

WAVES AND ELECTRONS

Published Monthly
by
The Institute of Radio Engineers, Inc.

VOLUME I — *January, 1946* — NUMBER I

Institute News and Radio Notes

Books

PROCEEDINGS Contents on page 1 P

Responsibility for the contents of papers published in WAVES AND ELECTRONS rests upon the authors. Statements made in papers are not binding on the Institute or its members.

The following editorials appeared in a preliminary issue of WAVES AND ELECTRONS in July, 1945. They remain pertinent and are accordingly here reprinted, essentially in their original form.

Waves and Electrons

A Publication of the I.R.E.

ALFRED N. GOLDSMITH, EDITOR

IN PLACING before members of The Institute of Radio Engineers and readers of its PROCEEDINGS of the I.R.E. this issue of a new publication, WAVES AND ELECTRONS, some historical and evolutionary information is properly included.

A third of a century ago, a small group of "wireless engineers" gathered with a common thought and intent. Their thought was that the nascent "wireless industry" had before it a future of untold promise; that there should be substituted in the corresponding technical field for mere trial-and-error and mysterious and confusing secrecy the frank expression of widely disseminated scientific truth; that to these ends an ethical attitude should be encouraged among the practitioners of the new profession of radio engineering; that the *common* good of the radio-engineering profession was not only the best course but also the most practical aim for farsighted workers in that field; and that to achieve these worthy and socially productive aims it was essential that there be formed a new radio-engineering society, strong in purpose, effective in execution, and worthy of the loyal and willing support of its members. Their intent was to form such a society and to give it their best thought and effort.

It was not an easy task that they voluntarily assumed in those days which are now so remote as to seem almost formless and incomprehensible to the modern and specialized worker in our field. Human reactions and frailties combined with material and financial difficulties to challenge the aspirations of those men. But the obstacles were ultimately surmounted and thus there was born our Institute of Radio Engineers.

Today, it is truly *The* Institute of Radio Engineers, the living embodiment of the professional aspirations of its members and their chosen medium for co-operative expression of their technical discoveries, plans, and proposals. Where once a handful gave their support to the Institute, there are now thousands; and thus The Institute of Radio Engineers in its fourth decade enjoys a membership far advanced into its second ten-thousand. It is world-wide in scope; fortunate in its organic unity with its many Sections in three countries; and properly encouraged by the work and standing of a national I.R.E. Council in a friendly country other than that of its origin. Its long-established publication is a recognized medium of consistently maintained standing. Its repute stands as high as its future seems bright.

For the latest decade, it has become clear that the original scope of the Institute has naturally expanded far beyond its former boundaries. The radio arts have been the ancestors of a host of technical applications of ever-increasing engineering and industrial interest and human significance. These later developments have come to be known by the generically descriptive term of "electronics"—a convenient though scientifically vague designation. Perhaps the most distinctive attribute of electronic methods is the utilization, somewhere in the corresponding systems, of electrons freed from material paths, and flexibly and instantaneously controlled to produce the desired effects.

The chosen publication medium of the I.R.E. members up to this time has been the PROCEEDINGS of the I.R.E. Its pages have literally been the history of the radio and allied arts. When its successive volumes are scanned, it is found that they have presented unobtrusively, but none the less comprehensively, the basic principles, devices, and methods of electronics as well as radio. Of late, this fact has appeared with ever-increasing clarity. It has seemed fitting therefore to present on the cover page of recent issues of the PROCEEDINGS a definite statement of the scope of that publication and the more detailed topical headings covering its usual contents. This accordingly has been done.

But another type of differentiation or evolution in the publication policy and procedure of the Institute now seems appropriate. This involves the addition to its publications, in one form or another, of an assembly of material of more general, historical, or tutorial nature and in some instances of particular and timely technical interest. This new material will also deal with the communication and electronic arts.

Communication of intelligence is essentially based on waves. Whether these be the pressure waves of touch and sound, or the electromagnetic waves of light and perhaps of neural messages, they constitute our basic means of communication and thus our essential contacts as individuals with the outside universe and with each other. Electronics, as stated, is functionally an expression of electron behavior and control. And, most remarkable of all, the electromagnetic wave and the electron seem to be but two aspects or modes of action of the same underlying and fundamental unity. What could therefore be more expressive of our cosmos and its contacts with humanity, as well as certain of the most powerful agencies now available for human communication and advancement, than the title of the new I.R.E. compendium: WAVES AND ELECTRONS?

In the hope and belief that The Institute of Radio Engineers will continue to be the primary repository and dissemination means of the engineering wisdom and accomplishments of its members in war and peace, and that its publications will ever be an agency promoting a brighter and more fruitful future for mankind, there is thus added to the activities of the Institute the issuance, according to an appropriate schedule, of additional material the title of which amply indicates the scope and means of the electronic and communication fields.

Electronics and Communications

Their Present and Future

W. L. EVERITT, PRESIDENT, 1945

COMMUNICATION engineering has ever been the originator of new applications of electricity and enterprises associated with it. The first commercial use of electricity was in the telegraph, and more than one electrical society was either established or fostered and encouraged by the early telegraph engineers. An early commercial application of electric currents generated by changing magnetic fields was the telephone of Bell, and the first applications of electronics were by Fleming and de Forest to the detection of radio waves.

Because communication engineering has many applications for small-current phenomena, it is usually the field in which new discoveries in electrical physics find their first application. Because it deals with such large ranges in power, impedance, and frequency, it requires a careful approach to the study of its phenomena to make sure that *conclusions* which may be drawn from particular experiments are based on an understanding of broad general principles.

All energy is transmitted by waves. When we apply a force (mechanical or electrical) to the driving end of a transmission system, the corresponding response is not felt immediately at the receiving end, but there is a finite time required for the effect to be propagated. If the force changes slowly, or if the transmission medium is stiff (as for instance in the connecting rod of a low-speed engine) the designing engineer pays little attention to the wave phenomena, or the finite time required for transmitting the disturbance. As the frequency is increased, approximations which neglect wave action are no longer valid.

In the early telegraph circuits, the transmission of direct currents over open wires seemed simple and straightforward. But as time went on, the telephone was invented, the lines became longer, cables with less stiffness were introduced, and higher frequencies were employed. Signal distortion became important, and a more complete theory of the travel of electrical waves along guided paths became necessary.

Coulomb studied the properties of electrons at rest (even though he did not know of their existence) and his law is the basis of our knowledge of electric fields. Ampere studied the properties of electrons in *motion:* and his law is the basis of our knowledge of the magnetic field produced by electric currents. Maxwell studied the properties of the acceleration of electrons and laid the basis for the initiation of those dynamic electric fields which do not require charges on which their lines of force terminate. He showed that the transmission of electric energy is not necessarily confined to the highways along which electric currents flow. Edison, and his successors in the electronic field, showed also that even electron currents are not necessarily confined to metallic highways. de Forest and Arnold showed the advantages for electronic traffic control of these nonmetallic gateways enclosed in the vacuum tube. From these foundations we see that our understanding of all electric phenomena is based on our knowledge of the principles connected with waves and electrons.

The war has shown what concentrated research and development and adequate financing may accomplish. The future of electronics and communications is bounded only by the ability of man to understand their principles and convert them to his uses. The engineer is essentially a builder and not a destroyer, and so the arrival of peace will stimulate him to even greater endeavors in the days to come if he is given the tools with which to work.

The primary function of The Institute of Radio Engineers is one of education. Since its organization the PROCEEDINGS has recorded most of the major advances in the electronic and communication art. It will continue to do so. As a result of the information dispensed in its pages many new developments have been initiated. The applications of these developments are spreading into many areas outside the field of communication. The expansion of principles, techniques, and applications are such that no longer can the individual engineer or scientist be well versed in them all. Therefore, a new educational function is required of the Institute in coordinating and summarizing the growth of knowledge and its application. To provide for this new function, either the recording of new advances must be restricted (which is unthinkable) or our publication activities must be expanded. I regard the advent of WAVES AND ELECTRONS as an historical event in the progress of the Institute whose significance will be recognized as the years go by and its contributions to the electrical arts expand and become of assitance to the professional workers in all the fields which it serves.

Kaiden-Kazanjian Studio

Benjamin E. Shackelford

Board of Directors—1945–1947

Benjamin E. Shackelford was born on August 12, 1891, in Richmond, Missouri. He received the A.B. degree in 1912 and the A.M. degree in 1913, both from the University of Missouri. In 1916, he received his Ph.D. from the University of Chicago.

From 1912 to 1914, Dr. Shackelford assisted in the physics department of the University of Missouri, and in the summer of 1925 was the first Brush Research Fellow at the Nela Research Laboratory. The following year, he joined the staff of Westinghouse Lamp Company, where his activities included work in illumination and incandescent lamp physics. His direct connection with radio began in 1916 when he undertook the engineering development of radio tubes for the company. He became manager of the radio engineering department in 1925, and his work with Westinghouse continued for approximately five years.

He became a member of the manufacturing department, radiotron division, of the Radio Corporation of America at Harrison, New Jersey, in 1930, and in 1934 was appointed manager of the patent department, activities of which included the operation of foreign technical agreements. After serving as manager of the company's foreign license service, Dr. Shackelford transferred to New York where he became assistant to the director of research and later to the chief engineer. In 1942, he was appointed engineer-in-charge of RCA's frequency bureau. Since 1944, he has been assistant to the vice-president in charge of RCA Laboratories, and manager of the license department of the RCA International Division with particular responsibility in the field of international activities.

Dr. Shackelford is a member of the American Physical Society, the Optical Society of America, the American Institute of Electrical Engineers, the Franklin Institute, the American Association for the Advancement of Science, Sigma Xi, Gamma Alpha, and Alpha Chi Sigma. He joined the Institute of Radio Engineers as an Associate in 1923, transferred to Member grade in 1926, and became a Fellow in 1938.

Preparation and Publication of I.R.E. Papers*

HELEN M. STOTE†

Summary—The various steps in the preparation of manuscripts for publication in the PROCEEDINGS of the I.R.E. and WAVES AND ELECTRONS as well as a short explanation of the handling of papers after they are received by the Institute, are outlined. The mechanical preparation of papers and the various stages through which they progress will be treated.

I. INTRODUCTION

TO SOME contributors to the pages of the PROCEEDINGS,[1] certain of the policies and procedures of the Editorial Department might seem either unnecessary or without sufficient justification. Yet, present methods of handling papers are based on long experience and on many requirements, some of which are not obvious. Further, it has always been the wish and desire of this Department to be logical, fair, and consistent in its treatment of all manuscripts submitted for consideration so that the best relations may exist between author and reader. This paper is presented in the hope that many questions which may confront or even puzzle authors may be answered.

Every publication has a broad underlying editorial policy which may differ radically from that of other periodicals, and even from those in the same field. Such policies promote coherence and consistency in style and form and thus reduce confusion or annoyance to the reader. Some few authors go so far afield from a desirable treatment of material that intensive editing of their papers is necessary (to say nothing of the required application, at times, of the simple rules of grammar and composition). Certain fundamental rules are also applicable to illustrations. It is obviously necessary that figures be presented in a clear form or the resulting printed page will lose in value to the reader.

It is one of the aims of the Institute to ensure the technical quality and originality of the papers appearing in its PROCEEDINGS. Accordingly, every manuscript received by the Editorial Department of the Institute is scrutinized by at least three members of the Papers Committee and one or more members of the Board of Editors before it is accepted for publication (or declined as unsuitable for the PROCEEDINGS). These readers carefully examine and study each paper and judge it on its likely value to the membership. Their reviews are of great benefit to the author whether the paper is finally accepted or not, for fundamental or less serious errors in manuscript are not infrequently caught, thus avoiding damage to the reputation of the author and impairment of the high quality of the PROCEEDINGS.

* Decimal classification: R053. Original manuscript received by the Institute, August 31, 1945.

† Publications Manager, The Institute of Radio Engineers, Inc., New York, N. Y.

[1] In this paper "PROCEEDINGS" means PROCEEDINGS of the I.R.E. and WAVES AND ELECTRONS.

II. THE MANUSCRIPT

Papers should be submitted in triplicate to the Editorial Department, The Institute of Radio Engineers, Inc., 26 West 58th Street, New York 19, New York. A complete set of illustrations should accompany *each* copy of the manuscript, since each paper is to be read by each of the three selected members of the Papers Committee. When three copies of the manuscript are submitted, they can go to the readers simultaneously. If only one or two copies are submitted, it is necessary to wait until they are returned to the Editorial Department before they can be forwarded to additional readers. This, obviously, delays the processing of the manuscript and the date of its publication.

The manuscript should be typed on white paper, regular letter size ($8\frac{1}{2} \times 11$ inches), double-spaced, and with margins about three quarters of an inch to one inch in width. Only one side of the paper should be used, and each page of every copy should be carefully checked to be certain that it is completely legible and without errors or omissions. Blueprint copies and silver-print copies are not desirable because, in the mechanical editing process, it becomes necessary to use a white pencil on the blueprints or a pen on the silver prints, and the clarity of the marked corrections leaves much to be desired.

The paper should have a short title, preferably not longer than five or six words. After the title, the name of the author should follow immediately, and a footnote reference should give the author's university, governmental, or business affiliation and location. If the paper has been presented at a meeting of any kind before any organization, this should be noted either in the footnote or in the letter of transmittal which accompanies the manuscript. The name of the meeting, the place, and the exact date should all be incorporated.

The author should state in his letter of transmittal of the manuscript any plans for the publication of his paper other than in the PROCEEDINGS. He should also inform the publicity division of his organization that it is the policy of the PROCEEDINGS not to publish any paper which has appeared in English in any publication having a substantial circulation among the readers of the PROCEEDINGS. The purpose of this policy is to avoid duplication of effort and waste of material, properly to conserve the funds of the Institute. Further, if the author is aware of any earlier partial or complete anticipation of publication in any language of the work described in his paper, he should state that fact early in his paper to avoid possible confusion or misunderstanding.

A short summary should precede the body of the

paper explaining the context in brief. Footnote references and equations should not appear in the summary.

It is desirable to have division headings, but to keep the number of these within reasonable limits. Subheadings are frequently confusing to the reader and should not be employed unless absolutely necessary.

At the end of the paper, all of the captions for the figures should be typed on a single sheet. Only the first word of each caption should be capitalized. The figures themselves should not carry these captions, but only an identifying figure number. It is highly desirable that the name of the author be on each figure. This identification is particularly helpful to the Editorial Department when the manuscript and illustrations are sent in separately, and also in identifying the proper paper when the illustrations are sent to the engraver to be made into cuts for the printer. Unrelated figures should not be divided into parts unless each part is an integral part of the whole figure. All figures should be numbered in chronological order according to the first reference made to them in the text.

The footnote references should give the full name of the author, the title of the paper, the publication in which it has appeared, the volume number, the beginning and concluding pages, and the month and year of the publication. As an example: Dorman D. Israel, "Looking forward in engineering education," Proc. I.R.E., vol. 23, pp. 353–355; June, 1945. References to books should give the name of the author, the title of the book, the name of the publisher, and the city, date, and year; where a particular page or chapter is referred to, this should be added. For example: V. K. Zworykin and G. A. Morton, "Television—The Electronics of Image Transmission," John Wiley and Sons, New York, N. Y., 1940, pp. 370–374. The editorial readers of the Proceedings feel that while in some cases a bibliography is desirable, generally speaking, it is better to indicate references within the body of the paper by means of footnotes which appear on the same page. In some cases, such as in an historical paper, a full bibliography is more desirable. Footnote references should appear in chronological order without exception. Numerals should be used in place of asterisks or daggers.

III. Illustrations

Wherever possible, illustrations should be submitted on sheets $8\frac{1}{2}$ inches by 11 inches (which is regular letter size), since the standard file cabinets used by the Editorial Department and others readily accommodate this size folder. Where larger sized paper is used, or where drawings are large and difficult to handle, there is the risk that both the manuscript and illustrations may be damaged in handling. Small illustrations or photographs may be lost if not attached to the $8\frac{1}{2}$- by 11-inch sheets. Extremely large illustrations are awkward to mail and are often a cause of trouble to the author and the publisher. (In some cases, the engraver makes an extra charge for the reduction of very large figures.) It should be kept in mind that most illustrations in the Proceedings are reduced to a maximum width of $3\frac{1}{4}$ inches. In some extreme cases, they are made larger, but in the interest of economy in paper and in cost, the sizes are kept to the named dimension wherever possible. Therefore, all printing and symbols on the illustrations should be of such a size that, when the figure is reduced to $3\frac{1}{4}$ inches or slightly less in width, the printing will be clear and distinct and the symbols legible. A suggested lettering size is given in the last paragraph of this section. Printing and symbols should be of the same size on all figures in any one group. Drawings should be done with black ink on tracing cloth or white paper. They should not be done with fountain-pen ink which may smear, fade, or reproduce badly. Where graph paper is used, it is desirable to use the kind which has a light-blue, cross-hatched background with the major divisions in red. In the photoengraving process, the blue drops out completely, and the red appears as black. (See Fig. 1.) Some authors submit figures on paper with yellow, green, brown, orange, or red cross-hatched paper. Such backgrounds reproduce in black with confusion and blurring of lines and lettering. (See Fig. 2.) A search was made for log-log paper with blue backgrounds and red major divisions, but the two leading companies which handle these papers do not manufacture log-log paper in these combinations. Therefore, where this paper is to be used, it is desirable to plot the chart on tracing cloth or white paper with only the major divisions indicated. Where this is impossible, and where the printing appears

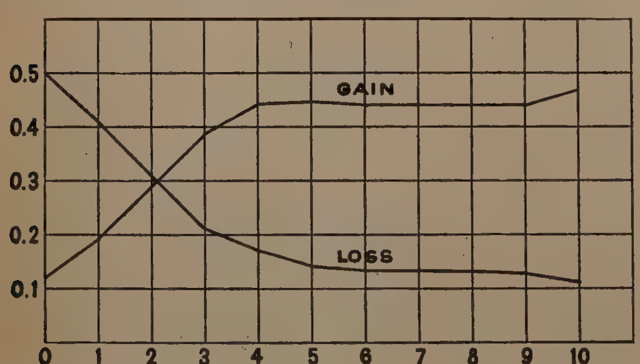

Fig. 1—Drawing made with black ink on blue cross-hatched paper with major divisions in red.

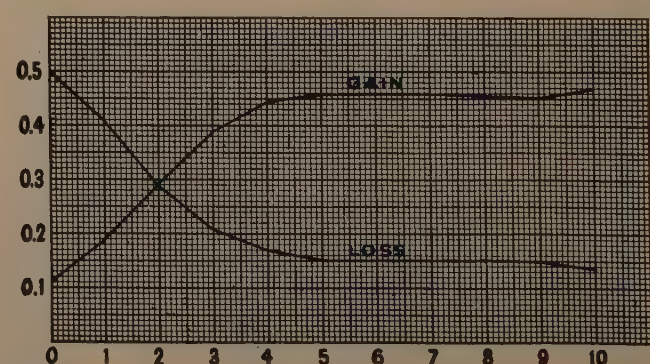

Fig. 2—Drawing made on cross-hatched paper with yellow, green, brown, orange, or red backgrounds.

on the figure itself, the author should copy the printing on small pieces of white paper and carefully paste them in the required places. When the cut is made, these letters will appear on little white "islands" and will be legible and sightly. However, one danger of this practice is that these small pieces of paper frequently fall off and are lost.

Photographs must be sharp in detail and on glossy paper. While matte-finished pictures are more artistic, much detail is lost in the photoengraving process. Any printing which necessarily is included on a photograph should be directly on it and not carried over into the white margin of the picture. The reason for this procedure is one of economy, for unless this method is used, the corresponding cut has to be made, in photoengraving terms, as a "combination half tone," and the cost is approximately twice that of an ordinary half tone.

Some commercial companies have a practice of printing a file number on the illustrations. Wherever possible, this is eliminated from the finished production; if it is impossible to "crop" it from the picture, it is necessary to have an artist airbrush it out. This extra work, of course, adds to the time element and to the cost of the cut, and is to be avoided wherever possible. Sometimes, holes are punched in the photograph itself, and these, too, are eliminated from the figure when the cut is made; or, where they interfere with the illustration itself, it is necessary to have the artist patch these holes. This patching process also adds to the cost of the cut and is not desirable. At present, the Institute does not employ an art staff to handle such special matters. Accordingly, it is necessary to send out all such work to be done by an artist. Sometimes, authors send in a rough pencil sketch. In these cases, it is necessary to return the drawing to the author for redrawing in an accepted form, and this, of course, slows up publication. Illustrations should not be mounted on heavy bristol board since such material is awkward to mail and greatly increases the postage cost.

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Some authors are so familiar with their own handwriting and with the symbols in the equations that they do not make these as clear or definite as could be desired. The author should, therefore, take extreme care in writing his equations so that there will be no doubt of his meaning in preparing the manuscript for use by the printer and typesetter. To avoid possible confusion, it is well for the author to write out the name of any new or unusual symbol on the margin of the paper. In the case of complex mathematical equations, he should insert parentheses, braces, and brackets where needed.

The equation number should be enclosed in parentheses and put at the far right of the equation. Equations should be numbered consecutively. Some authors start to renumber their equations with each section, or with each appendix. This practice is confusing to any later author who may use the paper as a reference.

Decimals should be preceded by a zero, such as 0.04, rather than .04.

V. Abbreviations

In every field, certain abbreviations and even slang terms are used. These are quite right and proper in their ordinary usage, but are believed to be unsuitable for the pages of the Proceedings. Slang and "shop talk" therefore, should not be employed. All words should be written out, (such as decibels, microvolts, megohms, and the like). The underlying reason behind this procedure is one which, until recently, was less important than usual, but is nevertheless fundamentally sound. The Institute of Radio Engineers is an international organization and has members in every part of the world who derive great benefit from the Proceedings. Many of these members have a reading knowledge of English, but may not be familiar with abbreviations or peculiarly idiomatic or popular expressions, and therefore may become confused in reading papers published in these pages unless they are formally presented. For the benefit of our foreign readers and our younger student members, the Editorial Department adheres to the strict policy of writing out abbreviations. The only exception to this is that, in tables, abbreviations occasionally may be used. Also, because of the long-established custom in all textbooks and journals, figures are always referred to as "Fig. 1," "Fig. 2," etc.

VI. Contributors Section

In the Contributors section, there are published formal portraits of authors. An attempt is made to keep the portraits reasonably uniform in size so that the page may present a pleasing appearance. The accompanying biographical sketch should include the author's name, date and place of birth, degrees received and dates awarded, institutions which granted them, places and dates of employment, and his affiliation with other professional, honorary, and scientific societies. A careful reading of several published biographical sketches should prove helpful in the preparation of this material.

VII. Processing a Manuscript

The author may wish to know what happens to his manuscript after it has been sent to the Proceedings for consideration.

As stated, the manuscript preferably is submitted in triplicate, and three sets of figures (blueprints, photostats, etc.), should accompany it, together with a complete set of original illustrations. The author should be certain that the manuscript is complete and he should not send additional material to be inserted later. Receipt of the manuscript is acknowledged to the author by the Editorial Department; a card is made out listing the name of the author, the title of the paper, and various other data, which card is kept active and up to date in the Editorial-Department files until final disposition has been made of the paper. A form is filled in and sent with one copy of the manuscript to the Editor for inspection. He examines the manuscript and selects the readers on the Papers Committee and the Board of Editors to whom the manuscript should be sent for review. The readers on the Papers Committee are skilled and recognized specialists in the field covered by the paper, and the member of the Board of Editors has a broad, over-all knowledge of both the field of the paper and the value of the submitted material to the Proceedings. Every attempt is made to select a variegated and impartial group of readers.

If the Editor finds that the publication of the paper might prove detrimental to the national interest, he sends the manuscript to a suitable referee or a government department for a decision in the matter. If, in the opinion of the reviewer, the manuscript falls into this class, it is returned to the author with such a notation.

When the paper appears on the desk of the Technical Editor, it is routed to the three designated members of the Papers Committee for review. When these three reviews have been received by the Editorial Department, they are sent, together with one copy of the manuscript, to the designated member of the Board of Editors. He then gives his opinion, and the whole file is sent to the Editor. The preceding steps are carried out by the Technical Editor. According to the reports of the readers, the Editor has three courses open to him: to accept the paper as it stands; to return the paper to the author for revision; or to classify it as unsuitable for publication in the Proceedings.

When a paper is accepted, the author is so informed, and any missing material is requested, such as a summary, original illustrations, captions for the figures, and a biographical sketch and photograph of the author. Where the author has previously had the last two items published in the Proceedings, he is informed of the date of their publication and requested to submit any changes which he desires to have made.

The paper is then mechanically edited. The type sizes are indicated for the printer, footnotes put in order, figures prepared for the photoengraver, and cuts made. The engineering correctness of the language and equations is carefully checked by the Technical Editor. The manuscript is edited for spelling, and the grammar is carefully checked. The Editorial Department of the Proceedings endeavors to make each paper as nearly grammatically correct as is possible, and relies for this purpose on the Second Edition of Webster's New International Dictionary, Unabridged; the Century Collegiate Handbook; and the Manual of Style, published by the University of Chicago Press. To any reader of the Proceedings, it is evident that hyphens are abundantly used. This conforms to the rules laid down by the previously mentioned books and is not arbitrary. Likewise, the usage found in the Proceedings of writing non-, semi-, super-, etc., words solid is in accordance with the spelling found in Webster's dictionary.

When the manuscript has been edited and the cut proofs checked against the original figures and found to be in good order, the paper is then sent to the printer. He sets it in type and returns a galley proof to the Editor, the Technical Editor, the National Bureau of Standards, and two copies to the author. The author inserts his corrections on one of the proofs and returns it to the Editorial Department, retaining the other copy for his files. After the galleys have been read by the Editor, Technical Editor, Department proofreader, the author, and the printer, all corrections are made on one master galley to which is added the decimal classification assigned by the National Bureau of Standards. This classification is inserted in the first footnote of the paper, which carries an asterisk instead of a numerical designation. It is not necessary for the author to trouble himself with the decimal classification.

The scheduling of papers for publication is carried out systematically to produce balanced, instructive, and interesting issues of the Proceedings. When this has been done, all of the manuscripts scheduled to appear in that particular issue are returned to the printer, together with the additional material which appears in the section of the magazine following the technical papers. This material consists of the Contributors section, book reviews, and miscellaneous items of interest to readers of the Proceedings. The printer makes all of the necessary corrections and returns page proofs to the Editorial Department. The Associate Editor and Technical Editor check the page proof, and the Editorial Department painstakingly goes over the dummied and corrected material, comparing it with the corrected page proof submitted. All corrections are checked, and final errors are noted for the printer. This material is then returned to the printer; after he has made the additional corrections and changes, the issue is ready to go to press.

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IX. Conclusion

It is hoped that the above discussion will clarify many problems in the minds of our authors, whether they have already contributed to the pages of the PROCEEDINGS or are contemplating writing a paper. The author should bear in mind that, in addition to his paper being technically sound, it is most desirable that the manuscript be presented in a clear and concise manner; and that, physically, its appearance be attractive, (that is, clean typing on white paper, wide margins, and equations carefully written or typed), figures in good order, and footnote and figure references presented chronologically. The paper is then far easier to review and does not present a later mental hazard to the PROCEEDINGS readers. It is reasonable to presume that a carefully prepared paper, well presented, enables more prompt action in the editorial processes than one less carefully prepared.

If the prospective author will carefully study an issue of the PROCEEDINGS, he will note the results of the procedure here described. A little thought and time spent on such a survey will likely be valuable to him and, incidentally, will greatly assist the editorial readers and the Editorial Department.

The Editorial Department is at all times eager and anxious to co-operate with the author, and any questions which he may wish to ask will be answered as promptly as possible. It is deeply appreciative of the friendly and effective help of many of the authors. It exists to work for the authors and the readers and the PROCEEDINGS of the I.R.E. and WAVES AND ELECTRONS is their magazine. The Editorial Department is here for no other purpose than to serve them.

An Ultra-High-Frequency Radio Range with Sector Identification and Simultaneous Voice*

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***Summary*—The primary purpose of a radio range for aircraft use is to provide a reliable indication to the pilot of an airplane as to his location with respect to a predetermined course. In addition, it is very desirable to identify quickly and positively the sector in which the airplane is at any given time; i.e., whether it is east or west of an east-west radio-range station. Voice radiated omnidirectionally is also desirable for ground-to-plane communication.**

The basis of the radio-range design described herein is the two-course localizer used in instrument landing. A group of three loop radiators provide two overlapping mirror-image patterns modulated at 90 and 150 cycles, respectively. A cross-pointer instrument, the vertical pointer of which is actuated differentially by the 90- and 150-cycle modulation, provides the pilot with the necessary information for orienting his plane.

A second pair of outside radiators, similar but at right angles to the first group, in conjunction with the center radiator, which is common to both the aural and visual systems, provides a keyed signal for aural sector identification. Except for the carrier radiation which is common to both the aural and visual signals, the two systems are entirely independent. Voice is radiated only by the center antenna.

The theory of the antenna system is discussed in this paper, paying particular attention to the problem of interaction between the aural, visual, and voice radiating systems. The various stages of development leading up to the final range installation at the Civil Aeronautics Administration Experimental Station in Indianapolis are given.

Introduction

THE PRIMARY purpose of a radio range for aircraft use is to provide a reliable aural or visual indication to the pilot as to his location with respect to a predetermined course. In addition, immediate and positive identification of the sector in which the airplane is at any given time; i.e., whether it is east or west of an east-west radio-range station, along with voice radiated equally in all directions from the station for communication purposes, is a very desirable feature.

The ultra-high-frequency radio-range with sector identification and simultaneous voice represents a highly specialized recent development to fulfill the above requirements. It seems desirable, therefore, to precede its

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description with a brief discussion of past developments in this field.

Four-Course Range and Two-Course Localizer

1. Four-Course Radio Range

The conventional four-course range, whether of the low-frequency type used throughout the country, or the ultra-high-frequency type, has a radiation characteristic similar to that of Fig. 1. Two mutually perpendicular figure-eight patterns are radiated successively, one keyed with characteristic identification A(· —) and the other N(— ·), the two signals being interlocked. The course is determined by the merging of the two interlocked 1020-cycle signals. A steady 1020-cycle tone informs the pilot that he is in the "on-course" region; a definite A(· —) or N(— ·) indicate the side of the course on which the airplane is flying.

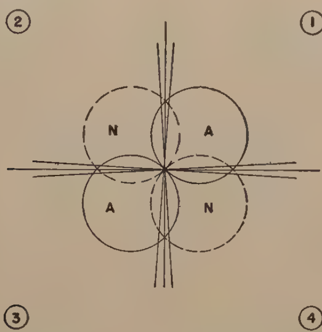


Fig. 1—Four-course radio range.

A limitation of this type of range is the identity of the "A" or "N" signals in opposite quadrants. As shown in Fig. 1, the same signal is received in quadrant 1 as in quadrant 3 (also quadrants 2 and 4). Information as to the airplane position with respect to the radio-range station is thus *not* conveyed to the pilot—a potentially serious cause of difficulty in case a pilot is lost and desires to fly to the nearest airport with a minimum of fuel consumption. The reader is referred to the indicated reference for further information on visual indication for orientation under instrument flight.[1]

2. Two-Course Localizer

With this localizer, the type installed at Indianapolis, Indiana, some five years ago and now commonly used for instrument landing, two characteristic patterns are radiated *simultaneously* rather than successively. One pattern is modulated at 90 cycles and the other at 150 cycles. The course is determined by an indicating instrument of the zero-center type actuated by the ratio of the 90- and 150-cycle modulations. When this ratio is unity the pointer position is in the zero-center of the instrument scale, thus informing the pilot that he is "on course." Predominance of 90- or 150-cycle modulation causes the pointer to swing to the right or left, respectively, indicating "off-course" flight.

Despite the practicability of establishing a course at least three times more sharply defined than with the four-course range, the difficulty of locating position with respect to the radio-range station remains. This will be

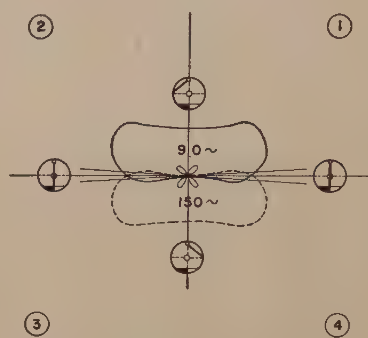


Fig. 2—Two-course localizer.

evident from Fig. 2; the same signal is received regardless of whether the airplane is in position "1" or "2" ("3" or "4"). The problem thus presented for solution was the identification of the sector at any instant so as to acquaint the pilot not only with his position with respect to the course (information which both the four-course radio range and the two-course localizer provide) but also his location with respect to the radio-range station.

Essential Range Requirements

Fig. 3 shows schematically the basic requirements and might represent any radio range, such as that at Indianapolis. Two indications are necessary: (1) deviation from

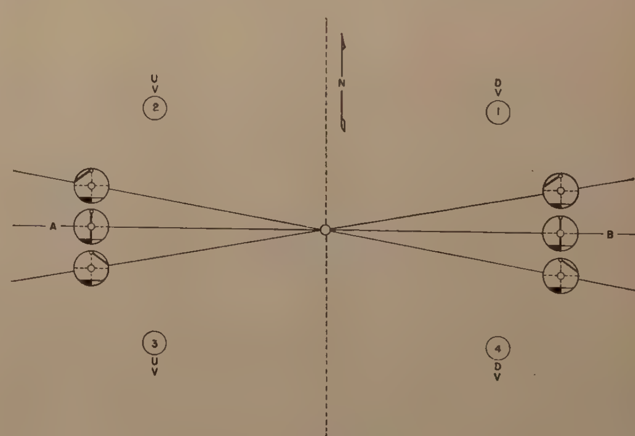


Fig. 3—Two-course radio range with sector identification and simultaneous voice.

the established course, provided visually by a zero-center type indicator which goes off scale approximately 10 degrees each side of the course; (2) aural sector identification; i.e., indication of the airplane position east or west of the radio-range station. In Fig. 3, the letter *V* represents voice which is radiated equally in all directions about the station.

[1] P. H. Redpath and J. M. Coburn, "Air Transport Navigation" Pitman Publishing Corporation, New York, N. Y., 1943, chapter 21, pp. 472–482.

Method of Solution

The solution of the signal problem of the two-course range with sector identification and simultaneous voice is indicated in Fig. 4. It is apparent from the preceding discussion that the two-course localizer provides a partial answer. Hence, two overlapping radiation patterns,

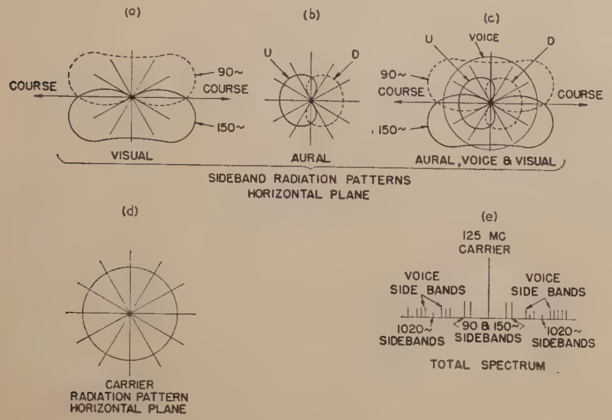


Fig. 4—Components of complete radiation.

modulated at 90 and 150 cycles, respectively, are transmitted simultaneously for the establishment of the east-west visual course (Fig. 4(a)). In addition, for aural sector identification, two radiation patterns are transmitted in immediate succession with interlocking D(— · ·) and U(· · —) characters; the first predominantly towards the east, and the second predominantly towards the west as in Fig. 4(b). Simultaneous voice, when applied, is radiated in a substantially circular pattern as illustrated in Fig. 4(c).

The complete radiation, aural, visual, and voice, is shown in Fig. 4(c), the relative sizes showing approximately the relative amplitudes of the aural, visual, and voice signals. The discussion concerning radiation patterns thus far refers to the sidebands only. By a process to be discussed shortly, the carrier which is common to the aural, visual, and voice signals is radiated in all directions as in Fig. 4(d). Under these conditions, Fig. 4(e) represents the total useful spectrum of the complete radio range.

Antenna Problem

The visual course radiations are produced by a group of three ultra-high-frequency loop antennas. These three antennas lie on a straight line perpendicular to the visual course with equal spacings between adjacent loops. The aural course radiators constitute a similar array oriented 90 degrees with respect to the visual group, the center loop being common to both groups. Voice, visual, and aural sidebands and carrier are radiated circularly only by the common center loop. The loop antennas used are the type previously described.[2]

[2] A. Alford and A. Kandoian, "Ultra-high frequency loop antennae," *Trans. A.I.E.E.*, (*Elec. Eng.*, 1940) vol. 59, pp. 843–848; 1940; and *Elec. Comm.*, vol. 18, pp. 255–265; April, 1940.

The antenna-array problem is illustrated schematically in Fig. 5. $S°$ represents the spacing of the loops in electrical degrees. With the amplitudes and spacings indicated the total radiation $F(\theta)$ in the horizontal plane is given by

$$F(\theta) = A \pm 2B \sin (S° \sin \theta). \qquad (1)$$

The choice of sign depends upon the relative phase of the outer loops with respect to the center loop and de-

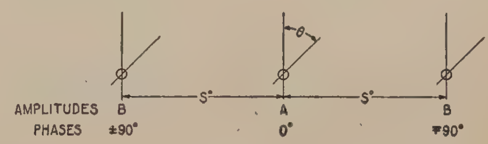


Fig. 5—Derivation of three-loop radio range.
$S°$ = spacing between loops in electrical degrees
Radiation pattern—horizontal plane: $F(\theta) = A \pm 2B \sin (S° \sin \theta)$
Visual antenna array: $A = 1$; $B = 0.707$; $S° = 135$ degrees
Aural antenna array: $A = 1$; $B = 0.5$; $S° = 100$ degrees

termines the particular image pattern obtained; the intersection of the two mirror-image patterns along direction $\theta = 0$ degrees and 180 degrees determines the

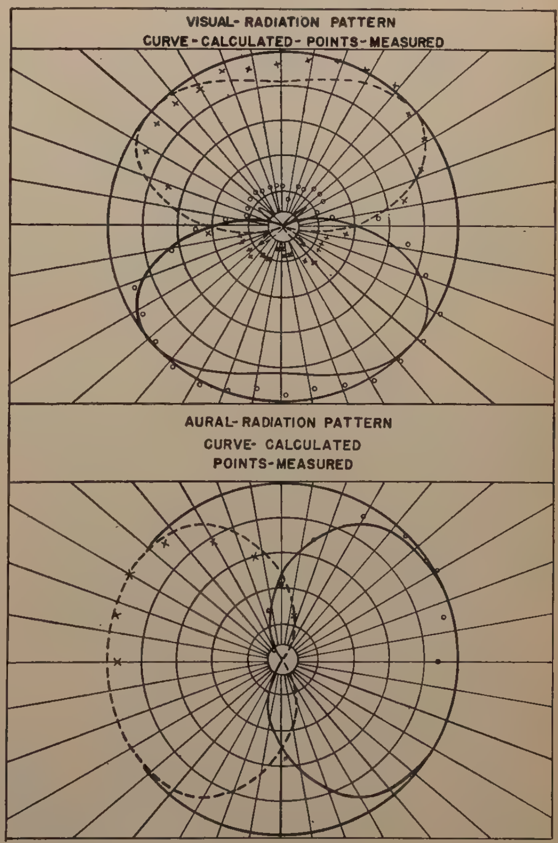


Fig. 6—Comparison of visual- and aural-radiation patterns.

established course. The first term A represents the radiation from the center loop; the second term, the radiation from the outer loops.

Equation (1) is generally applicable to overlapping radiation patterns. In this form, or slightly modified and

expanded, it may be applied to a variety of radio-range and localizer antenna arrays.

For the case of the visual and aural arrays, the requirements impose a division of power between the center and outer loops of $A=1$ and $B=\frac{1}{2}$ with $S=120$ degrees. These values give an infinite clearance at angles of ± 48.6 degrees to either course, where clearance is defined as the ratio of the field strength of one mirror-image pattern to the other at any selected distance from the station.

Fig. 6 shows the visual- and aural-sideband radiation diagrams for the values of A and B indicated in Fig. 5. This choice of current ratios and spacings is not the one used in the final range installation, but has been included to illustrate the effect of antenna spacings and current ratios on the radiation patterns.

1. Aural Array

The circuit of the aural antenna system is illustrated in Fig. 7. By keying at the indicated location, the phase of the two outer loops is reversed with respect to the phase of the center loop; hence, the desired mirror-image patterns are obtained alternately. In one radio-frequency keyer position, radiation occurs predominantly

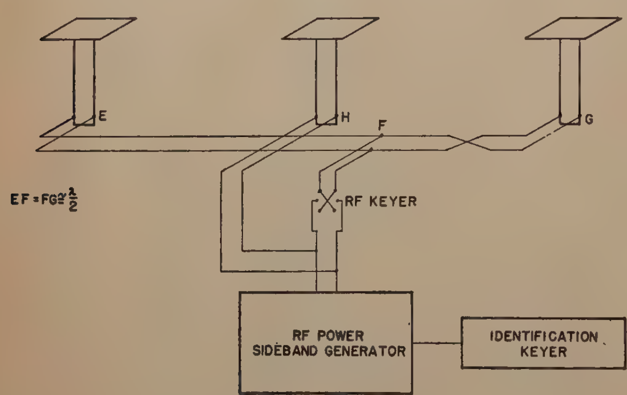


Fig. 7—Aural antenna network.

toward the east and the keyed identification is D. In the opposite position, radiation occurs predominantly towards the west and the keyed identification is U. Two separate antenna systems for radiating the two characteristic patterns consequently are avoided, inasmuch as the separate patterns are obtained at will by a simple phase reversal.

In this type of array, the problem of interaction between the various radiators must be considered. Since the outer loops have equal currents of opposite phase they do not induce any current in the center loop. The center loop, however, may excite the outer loops parasitically. This parasitic action can be controlled and made useful for certain applications. In the present case, however, it is undesirable and hence the relationship $EF = FG \cong \lambda/2$ is maintained (Fig. 7). This places a virtual short circuit at terminals E and G for parasitic currents, and detunes these loops insofar as parasitic action is concerned.

2. Visual Array

Fig. 8 depicts the visual antenna system which is similar to the aural array except that its position is oriented 90 degrees from the visual position. In this case, however, the two characteristic patterns must be transmitted simultaneously, rather than successively, so that a special network is required.

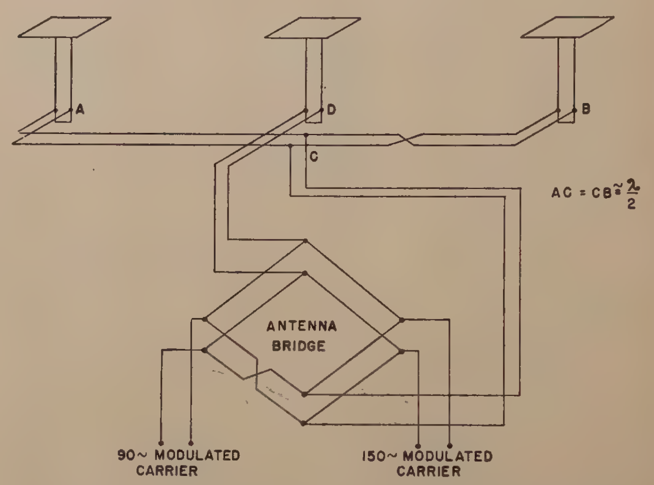


Fig. 8—Antenna network, two-course visual radio range.

This network, a transmission-line bridge, is indicated diagrammatically in Fig. 8. The 90- and 150-cycle modulations are fed into opposite terminals of the network and arrive at the center loop in phase; but because of the phase reversal in the arm of the bridge, they reach the outer loops in opposite phase. This results in the simultaneous mirror-image patterns, one with 90-cycle and the other with 150-cycle modulation.

The bridge circuit possesses another important advantage. At each of its two input terminals, in addition to the 90-cycle and 150-cycle sidebands, there is present the 125-megacycle carrier, which arrives at the respective input terminals of the bridge in phase. At the terminals leading to the outer loops, however, the carriers cancel out because of the previously mentioned phase reversal in one arm of the bridge. Contrariwise, at the center-loop terminals, the carrier is in phase and is therefore additive. Hence, no mirror-image patterns exist insofar as the carrier is concerned; the carrier is radiated only from the center loop equally in all directions. The sideband power, however, divides equally between the center and the outer antennas and makes possible the radiation patterns already described.

The transmission-line bridge thus serves three highly important functions: (a) the realization of two different radiation patterns from a single antenna array; (b) the removal of the sidebands from the carrier of two modulated waves of the same carrier frequency without any power dissipation; and (c) the radiation of the total carrier energy solely from the center loop with uniform circular distribution.

The Visual Modulation Problem

The problem of obtaining two equal sources of radio-frequency power modulated at 90 and 150 cycles, respectively, requires careful consideration. Early in the development of radio ranges it became evident that two separate transmitter output stages, each with its own modulation, was not satisfactory. This was due to the fact that variation of one output stage with respect to the other resulting from a change in tube emission, or any other reason, would alter the established course correspondingly. It was necessary, therefore, to divide the transmitter carrier output into two equal parts and modulate each half separately by means not subject to difficulties arising from change of tube emission. Mechanical modulation, consequently, was adopted.

The schematic diagram of the mechanical modulator is shown in Fig. 9. It will be seen that the transmitter

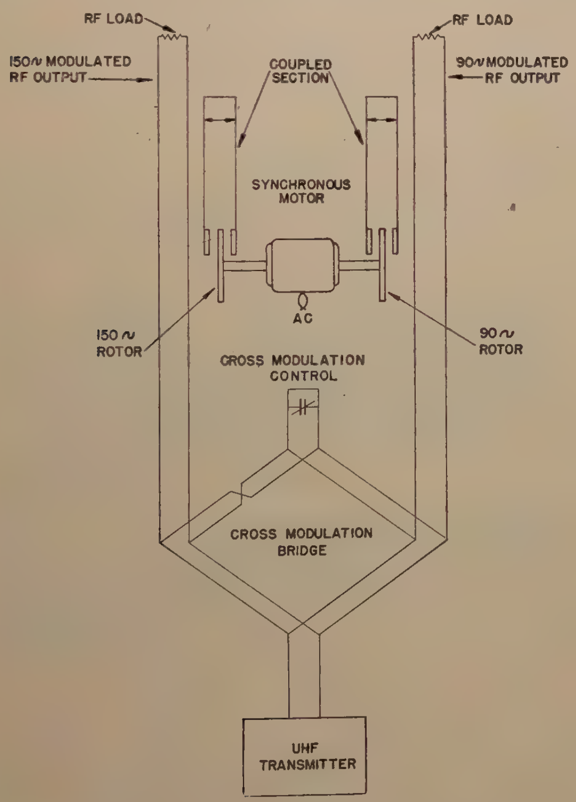
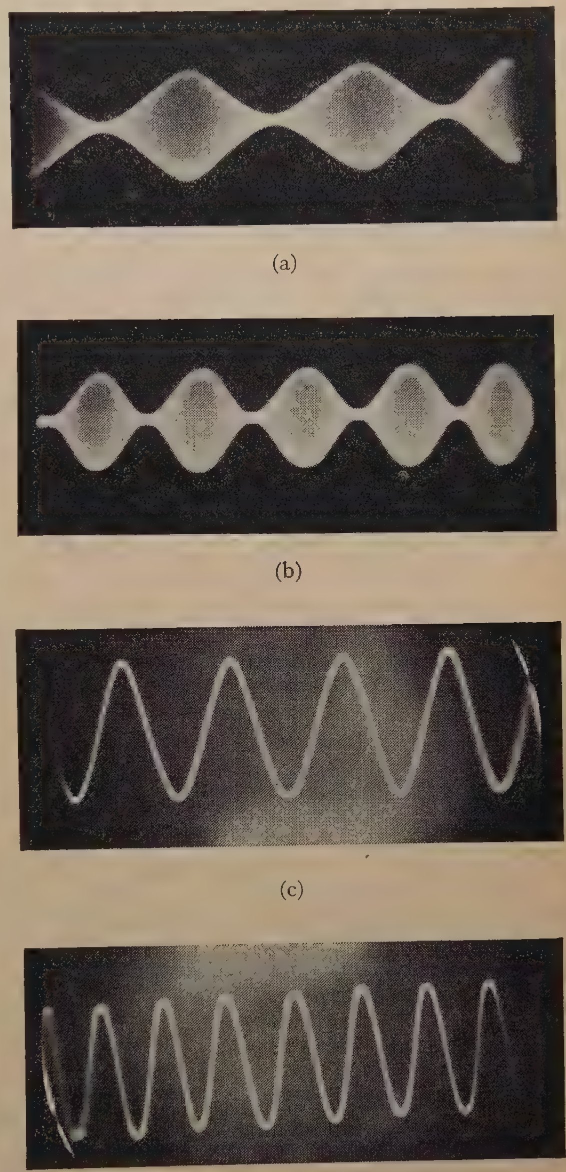


Fig. 9—Ultra-high-frequency mechanical modulator.

output is divided into two channels with a resonant quarter-wave section coupled to each. Under these conditions, the coupled sections effectively short-circuit the transmission line to the antennas.[3] These resonant sections are detuned periodically by 3- and 5-blade paddle wheels rotating at 1800 revolutions per minute. Thus the resulting output of each channel is modulated 100 per cent. By tuning the sections so that resonance is approached but not reached, any lesser degree of modulation may be obtained.

In arbitrarily dividing the output of a transmitter into two channels, special precautions are required to prevent cross modulation when the impedance of a channel varies during the modulation cycle. For this purpose the transmission-line bridge again proves to be



(a) (b) (c) (d)

Fig. 10—Wave form from mechanical modulator.
(a) 90-cycle modulation radio-frequency envelope
(b) 150-cycle modulation radio-frequency envelope
(c) Detected 90 cycles
(d) Detected 150 cycles

a versatile tool. By varying the impedance at the terminal opposite the transmitting end, it is possible to make each branch entirely independent of the other, and hence obtain substantially zero cross modulation. It is, moreover, not difficult to show experimentally that no power need be lost in the bridge terminating network to obtain negligible, less than 1 per cent, cross modulation between channels.

[3] Andrew Alford, "Coupled networks in radio-frequency circuits," Proc. I.R.E., vol. 29, pp. 55–70; February, 1941.

The tendency prevails to associate mechanical modulation with jagged, distorted, or at least square-wave modulation rich in harmonic content. In the present case, however, this type of wave configuration is decidedly not obtained, as will be evident from Fig. 10, showing representative results. It is, in fact, not difficult to limit the total distortion of each channel to less than 10 per cent and, with somewhat more care, to less than 5 per cent.

Aural and Voice Modulation

The present method for application of aural and voice modulation is a result of several years development and experimentation. Throughout these years of growth leading to the present radio range, several methods were employed to achieve this addition. It seems logical, therefore, to describe the three major methods in the order in which they occurred.

Method I

Initially, only aural modulation without voice was used to obtain sector identification. In the adaptations

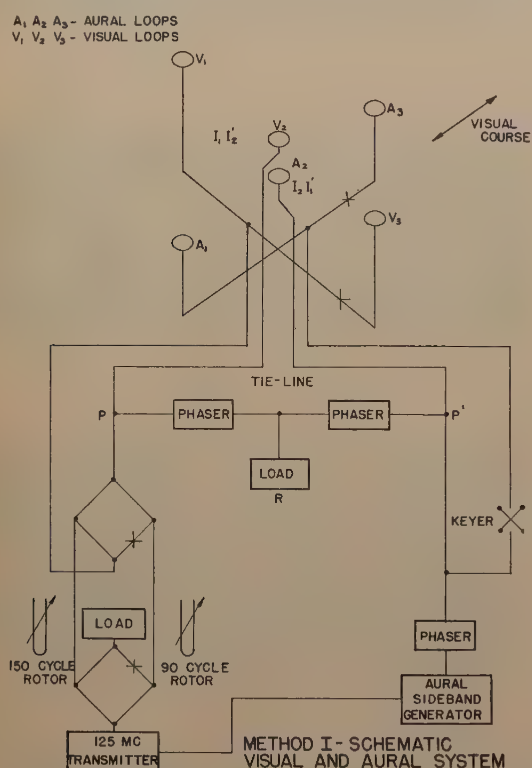


Fig. 11—Method I, schematic visual and aural system.
A_1, A_2, A_3=aural loops
V_1, V_2, V_3=visual loops

to be described later, voice is added to the total radiated spectrum.

It is evident if a separate carrier were used for the aural portion of the radio range, and the radiation directed by means of the radio-frequency relay first to the east, then to the west, the total carrier available at the receiver would fluctuate, and hence the automatic volume control of the receiver would be affected. This would cause "kicking" on the visual course-indicating instrument each time the aural signal was keyed. It follows, therefore, that some means must be provided to transmit only 1020-cycle sidebands for the aural signal and to make use of the already existing circularly radiated carrier from the visual-instrument course.

To accomplish these results, the aural-channel facilities employ a sideband generator giving an output predominance of sideband to carrier of 30 to 40 decibels depending upon the care exercised in adjustment. At the output of the sideband generator, a phaser is provided to obtain the correct phase relationship between the sidebands thus produced and the carrier from the main transmitter.

Since a carrier common to both the aural and visual modulation is utilized, it is not desirable to modulate the carrier in the mechanical modulator 100 per cent.

This modulation, therefore, is reduced to approximately 70 per cent and the aural signal then modulates the remaining 30 per cent.

Interaction Problem

Method I

The block schematic diagram of method I of the two-course radio range with sector identification is shown in Fig. 11. The visual and aural loops shown in this illustration are positioned above a metallic counterpoise.

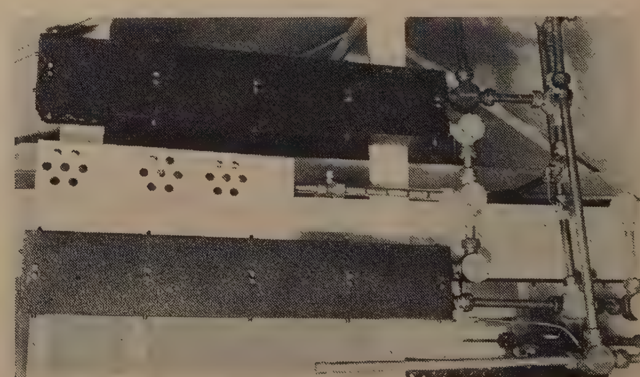


Fig. 12—Tie line used in method I to prevent interaction between aural and visual antenna systems.

The visual loops are placed a half wave above the counterpoise and the aural loops slightly more than one quarter. From previous discussion it is clear that the four outer loops induce no currents in the center loops. The center two loops, on the other hand, do not induce currents in the outer loops because these loops are detuned for parasitic current, since in Figs. 7 and 8

$$AC = CB = EF = FG \cong 180 \text{ degrees.}$$

Thus care is taken of all interaction, except that between the two center loops, one mounted above the other. The coupling between these two loops is serious because the visual signal will get into the radio-frequency relay in the aural circuit; furthermore, the aural sidebands would feed back into the mechanical modulator. Thus, a great deal of undesirable interaction

between the aural and visual systems would result.

These difficulties are overcome by means of a properly designed tie line such as the one shown in Fig. 11. The installed tie line is shown in Fig. 12. The general function of the line is to borrow power from the source, and control it in phase and amplitude so as to neutralize the unwanted voltage at a specific point.

The design considerations will be clear from the following: assume the directly fed current in the visual center loop V_2 (Fig. 11) is represented by I_1 and the induced current in the center aural loop A_2 is I_1'. If, now, a short circuit is applied along the feeder to the lower loop, it is evident that I_1' can be controlled in amplitude and to a certain extent in phase. In fact I_1' can be made negligibly small by placing the short circuit at the correct location P'. In practice, instead of an actual short circuit, a virtual short circuit is produced by means of the tie line. As a result, the top center loop is made substantially independent of the lower center loop. Conversely, point P can be located on the visual center-loop feeder in a manner such as to neutralize all the current induced in the top loop by the lower loop. The tie line, to be effective, requires a minimum of two controls: one for change of phase and another for amplitude. For phase control, some resistance is necessary in the circuit. Optimum design dictates very low power dissipation; but with negligible power loss, tuning of the line becomes critical. A power dissipation of approximately 10 per cent in the resistance load T (Fig. 11) results in adjustments that are readily made and stay put indefinitely.

Method I had the following disadvantages which resulted in its replacement by method II: (1) the tie line employed for the prevention of interaction between the center loops did not lend itself to adjustment by a maintenance man; and (2) the lower height of the outside aural loops above the metallic counterpoise gave rise to a radiated vertical component along the visual course. This vertical component occurred due to reradiation from a high current concentration induced in the metallic counterpoise below the center aural loop by the outside aural loops. In such an ultra-high-frequency radio range utilizing horizontal polarization, freedom from any vertical polarization is highly essential if the course is to be independent of the attitude of the airplane. If the course is dependent on the attitude of the airplane, a flight phenomenon known as "pushing" occurs, and the plane will zig-zag about the course in attempting to follow it.

Method II

A block schematic of the second method appears in Fig. 13. This differs from the previous method by the addition of voice to the system and the elimination of one center loop antenna and associated tie line.

The use of a bridge permits one antenna to be energized by two different sources without interaction between them. This ability of the bridge is utilized to excite a single center loop by both the visual and the combined aural and voice channels,[4] as illustrated in Fig. 13. The bridge B_1 also prevents either channel from feeding into the other.

Another bridge B_2 is used to apply voice. The voice facilities consist of an additional sideband generator and modulator. A phaser is provided at the output of each sideband generator to place the sidebands in the proper phase relation with the carrier at the output of bridge B_1 feeding the center antenna.

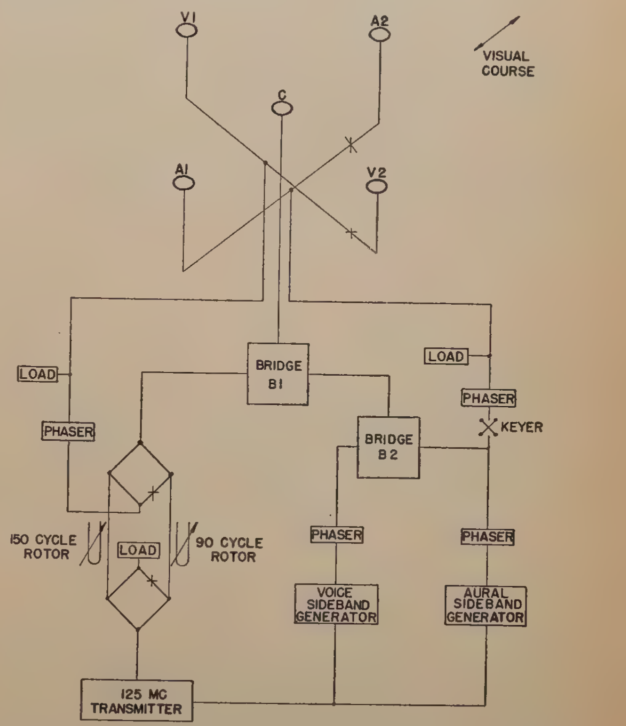


Fig. 13—Method II, schematic visual, voice, and aural system.
A_1, A_2, C=aural loops
V_1, V_2, C=visual loops
C=voice loop

The voice sidebands do not feed into the aural outside antennas, due to the action of the bridge B_2. Likewise, the aural sidebands cannot reach the output of the voice generator.

Initially, the voice sideband generator was modulated by a 20-kilocycle subcarrier. This 20-kilocycle subcarrier was modulated with voice. The voice facilities were later provided with a switching arrangement to permit an instantaneous change from voice on the subcarrier to voice directly on the main carrier. This permitted a close comparison during actual flight tests. With voice directly on the main carrier, a 1020-cycle rejection and a high-pass filter were provided in the voice channel to prevent any disturbance in the visual and aural courses with voice modulation. Flight checks showed equivalent results between the two methods of voice modulation.

The loads on the outside antenna feed lines shown in Fig. 13 are used to dissipate an amount of sideband

[4] The use of the transmission-line bridge in this manner was first suggested by W. E. Jackson, chief of the Radio Development Section of the Civil Aeronautics Administration.

energy necessary to give the proper power ratio in the outside loops to the center loop.

Method II had one inherent disadvantage. Half of the total carrier power was dissipated in the load terminating the bridge B_1. As a consequence, method II was replaced by method III.

Method III

The final system used utilizes an adaptation of the mechanical-modulator bridge arrangement to the aural and voice channels. The sideband generators used in the previous method have been replaced by two 35-watt radio-frequency amplifiers (No. 1 and No. 2), shown in Fig. 14. Amplifier No. 1 is modulated with voice plus

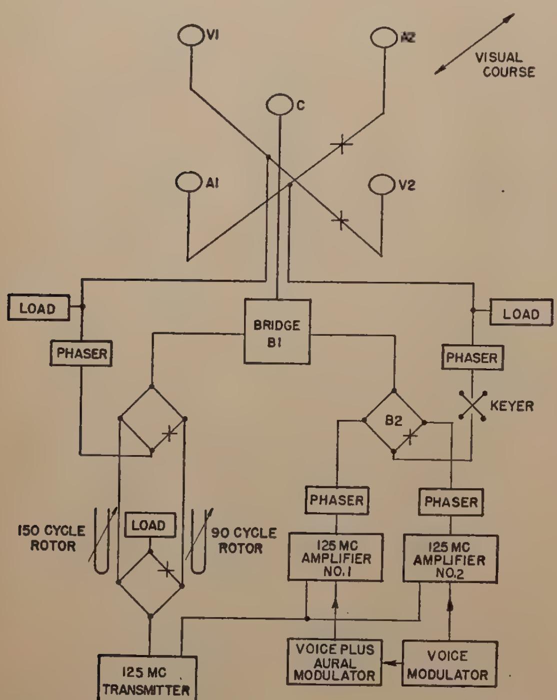


Fig. 14—Method III, schematic visual, aural, and voice system.
$V1$, $V2$, C=visual loops
$A1$, $A2$, C=aural loops
C=voice loop

1020 cycles, while amplifier No. 2 is modulated with voice only. The percentage of voice modulation on the equal carrier outputs of each radio-frequency amplifier is the same. As a result, the voice sidebands and carriers cancel out at the terminals of bridge B_2 feeding the outside aural antennas. The 1020-cycle sidebands of amplifier No. 1 divide equally between the center- and aural-antenna feed lines.

Since no crossover exists in the arms terminating in the upper junction of bridge B_2, the carriers and voice sidebands of both amplifiers add. The carrier power dissipation at bridge B_1 is now a small fraction of that dissipated in Method II. The sideband generators, which require careful adjustment, have been replaced by straightforward radio-frequency amplifiers.

The percentage modulation of each channel on the carrier is approximately 50 per cent for the visual, 35 per cent for the voice, and 15 per cent for the aural sidebands. These values were found optimum by actual flight tests.

125.020-Megacycle Marker

A problem which presented itself, when flight checks were begun on this radio range, was irregularity of pointer indication when the plane flew at high vertical angles with respect to the transmitting equipment. This was due to lack of directly radiated signals, since the loop antennas have substantially zero radiation vertically. The receiver in the plane, because of its automatic-volume-control characteristic, became very sensitive and picked up whatever stray signal existed and hence gave irregular pointer indication.

Several possible solutions were discussed with the Civil Aeronautics Administration personnel, and as a result, a special marker was used to overcome this difficulty. Fig. 15 shows the marker array which is fed from

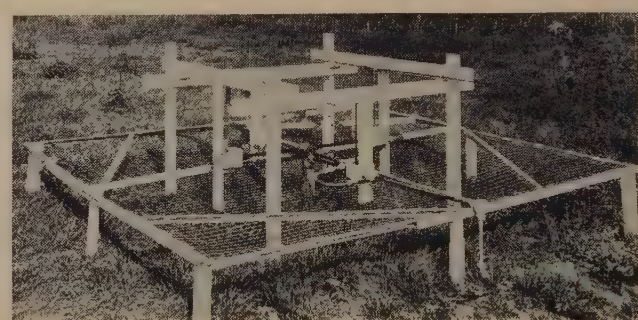


Fig. 15—125.020-megacycle marker antenna array.

an auxiliary 30-watt transmitter removed in frequency from the main transmitter by approximately 20 kilocycles. This signal has no modulation and serves merely to radiate carrier straight up in order to steady the cross-pointer indicator and silence the receiver in the airplane at high vertical angles over the radio-range station.

Equipment

The receiver, designed for this radio range, is a Western Electric type RUM crystal-controlled 125-megacycle superheterodyne with an intermediate frequency of 10 megacycles. A high-pass filter, above 150 cycles with 1020-cycle rejection, in conjunction with a 90- and 150-cycle pass filter and a 1020-cycle pass filter, were inserted in the audio channel to separate the aural and voice signals from the 90- and 150-cycle visual signals. A more recent receiver, the Western Electric type 32A, has also been used in flight checks with very satisfactory results. The course indication is provided by a Weston cross-pointer instrument, the vertical pointer of which is utilized as illustrated in Fig. 2. The cross-pointer instrument is used in conjunction with 90- and 150-cycle pass filters in parallel. The outputs of the filters are rectified to actuate the meter. With a predominance of 90-cycle modulation, the vertical pointer deflects to the right of its "on course" or center position, while a predominance

of 150-cycle modulation swings the pointer to the left. The receiving loop antenna on the airplane is similar to the type used for instrument landing.[5]

Figs. 16, 17, 18, 19, and 20 show views of the complete radio range, the transmitting equipment, antennas, and the airplane equipment comprised in the radio range.

Fig. 16—Two-course radio range with sector identification and simultaneous voice, showing transmitter house, counterpoise structure, antenna house, and 125.020-megacycle marker.

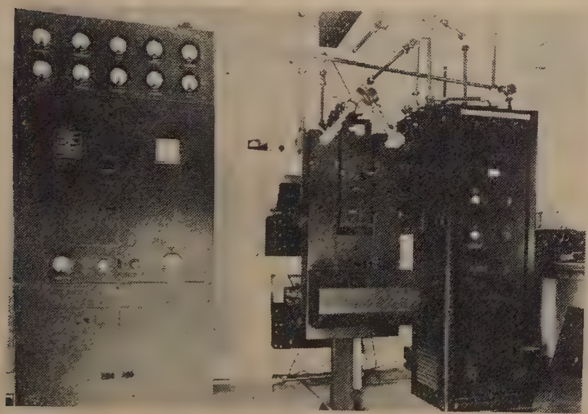


Fig. 17—Transmitting equipment, showing left to right, 125-megacycle, 300-watt transmitter, mechanical modulator, and sideband generator. Auxiliary marker transmitter is in the background.

Acknowledgment

The help and co-operation of the Radio Development Section of the Civil Aeronautics Administration and particularly the personnel of the Civil Aeronautics Administration Experimental Station of Indianapolis, Indiana, is gratefully acknowledged.

[5] P. C. Sandretto, "Principles of Aeronautical Radio Engineering," McGraw-Hill Book Company, Inc., New York, N. Y., 1942, chapter III, pp. 100–105.

Fig. 18—Visual and aural loop antennas mounted above metal counterpoise.

Fig. 19—Civil Aeronautics Administration Boeing used in flight checks, showing receiving loop antenna.

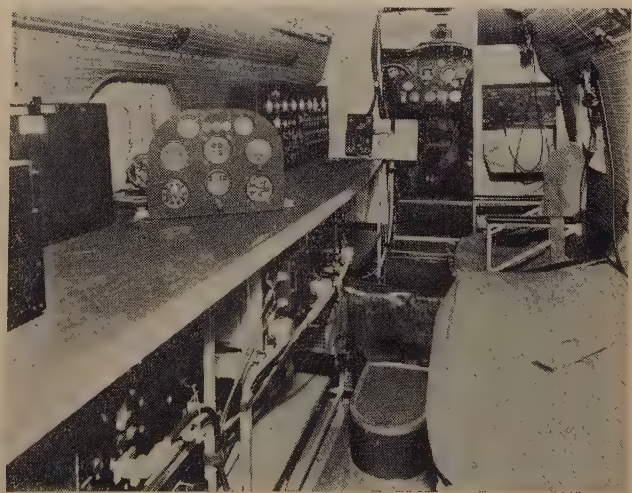


Fig. 20—Cabin view of Civil Aeronautics Administration Boeing ready for demonstration flights.

A Simple Optical Method for the Synthesis and Evaluation of Television Images*

ROBERT E. GRAHAM†, MEMBER, I.R.E. AND F. W. REYNOLDS†

Summary—**A combination of a 35-millimeter motion-picture projector and a line screen enables the projection of still or motion pictures closely similar in appearance to those produced by television. This similarity of appearance is checked theoretically by an analysis of the type previously reported by Mertz and Gray in a discussion of the theory of scanning. From the analysis it is shown that five parameters of the optical-simulation system may be varied to obtain the equivalent of variations in television factors such as number of scanning lines, size and configuration of scanning apertures, and width of frequency band.**

Photographs of simulated television pictures projected by this method are presented. These pictures include subject matter of general interest as well as selected subjects to illustrate the spurious detail components introduced by the television scanning process. These components produce moiré patterns, "steps" on diagonal lines, and impairment of vertical resolution. Simulation pictures projected by this method have been compared with those produced by a television system and the expected agreement observed.

Calculations are given of the diffraction effects in optical systems of this type and it is shown that the departure from geometrical theory is small in the arrangements described.

I. Introduction

IMAGE transmission systems are often compared in terms of frequency bandwidth, number of scanning lines, and picture repetition rate. Specification of these factors sets a theoretical upper limit for the system performance. However, a number of other variables exist which may greatly influence the image quality. Among these are picture brightness, over-all tone reproduction, and the admittance characteristics of the scanning apertures. It is possible to measure these quantities, but difficult to obtain a quantitative estimate of their effects upon subjective image quality. It is therefore desirable to make subjective studies of such factors, employing real or simulated television systems.

Optical-simulation methods have been found useful in conducting these studies and in providing reproducible standards of image quality. Several methods have been suggested and used for these purposes. Engstrom[1] has described a method of optical simulation which uses crossed pieces of lenticular film for subdividing an image into elemental areas or picture elements. Goldmark and Dyer[2] have described a mechanical scanning arrangement which produces television-simulation photographs. Baldwin[3] has used a motion-picture projector for simulating television images for direct viewing, the area of confusion being determined by the amount of defocusing and an adjustable aperture at the projection lens.

The method of simulation described in this paper employs an out-of-focus optical system in combination with a suitable line screen. Use of the latter enables an accurate reproduction of the spurious components which result from the strip-scanning process employed in television. It is well known that these components cause a definite loss in vertical resolution as well as occasional annoying patterns which are superposed upon the normal image. They appear as a step-like structure at oblique edges of object detail, and as moiré patterns when an array of object lines forms a small angle with the scanning direction. In previous simulation work[3] the effects of the spurious components upon picture sharpness have been evaluated and allowed for on the basis of subjective comparisons between out-of-focus simulation images and television images of known characteristics.

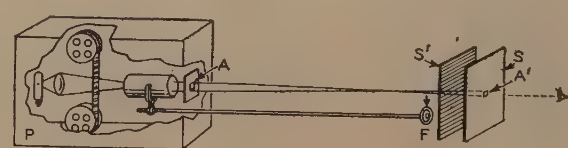


Fig. 1—Optical-simulation apparatus.

The theory underlying the line-screen method of simulation is developed at some length in this paper, and it is shown that the method is capable of accurately simulating the television process. Several examples of simulation are shown and discussed, including subject matter particularly selected to exhibit pronounced spurious patterns.

II. Description of Apparatus

A pictorial sketch of the equipment used in the simulating process is shown in Fig. 1. A 35-millimeter motion-picture projector P projects images from ordinary film on the ground-glass screen S. A line screen S' consisting of a parallel array of alternate lines and spaces is interposed between the projector and the screen S. The projected image is focused a given distance on the near side of S' by means of the focusing control F, which

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† Bell Telephone Laboratories, New York, N. Y.

[1] E. W. Engstrom, "A study of television image characteristics," Proc. I.R.E., vol. 21, pp. 1631–1652; December, 1933.

[2] P. C. Goldmark and J. N. Dyer, "Quality in television pictures," *Jour. Soc. Mot. Pic. Eng.*, vol. 35, pp. 234–253; September, 1940.

[3] M. W. Baldwin, Jr., "The subjective sharpness of simulated television images," *Bell. Sys. Tech. Jour.*, vol. 19, pp. 563–587; October, 1940.

controls the position of the projector lens. This results in an out-of-focus image on S, each subject point giving rise to a uniform[4] area of light A', which is a shape replica of the projector aperture A.

The control elements of the apparatus are: (1) the degree of out-of-focus of the projected image; (2) the number of lines in the screen S'; (3) the configuration of the aperture A; (4) the positioning of the line screen between the projector and S; and (5) the nature of the opacity variation found by traversing the alternate lines and spaces of S'. These factors collectively determine the simulated television picture.

Briefly, the significance of these five factors is as follows: (1) determines the general resolution level of the picture; (2) fixes the number of scanning lines in the equivalent television system; (3) determines the configuration of the television scanning apertures; (4) provides an adjustment for controlling the nature of the spurious components; and (5) determines the amplitude of the spurious components.

The pictures appearing in this paper were, except as noted, taken under the following conditions. The projector lens had a focal length of 6 inches and was used with a square aperture approximately 0.7 inch on a side. The projected image was 8 by 10 inches and was formed at an image distance of about 6 feet. The line screen contained approximately 400 black lines across the 8-inch dimension.

In projecting still pictures with the apparatus, it was found desirable to insert a water cell between the film gate and the light source to avoid excessive heating of the film. Also, film buckling was minimized by mounting the film between glass slide covers. In order to obtain a uniform light distribution across the projector aperture A, a ground-glass diffusing screen was inserted behind the film gate. For direct comparison of the simulation pictures with actual television pictures, a color match was desirable. This was attained by placing appropriate color filters behind the film gate.

III. Theory of Optical Simulation

The manner in which the optical system of Fig. 1 makes possible an accurate television simulation may be brought out by developing a mathematical expression for the transformation of the initial picture field into the final image field as seen at S. This expression will be compared with a like expression for the transformation of the subject picture produced by television systems. The similarity of result obtained in the two cases will be used to support the validity of the simulation method.

A diagrammatic sketch of the optical system is shown in Fig. 2. On the basis of geometrical theory, the light from the point of the film plane on the optical axis passes through the lens, is restricted to a ray bundle bounded by the aperture A, converges to a point on the axis at the sharp-focus plane S'', and then diverges to a light spot A' on the screen S. The "confusion area" A' has the same boundary shape as the aperture A, but does not contain a uniform distribution of light because of the striation introduced by the line screen S'. Similarly off-axis points in the film plane are refocused in the plane S'' and give rise to confusion areas at S similar in shape to the aperture A.

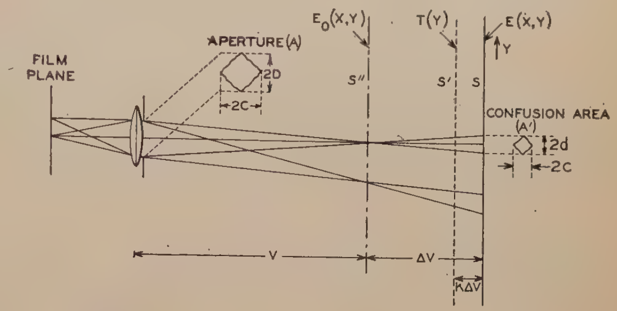

Fig. 2—Diagrammatic arrangement of optical-simulation system.

Setting up a system of axes with the y axis vertical, the x axis perpendicular to the plane of the drawing, and the origin on the optical axis; the intensity of illumination falling on the S'' plane may be written as $E_0(x, y)$. This function represents the effective brightness distribution in the subject picture, and thus may be treated as the source picture field. The illumination intensity at S may be written as $E(x, y)$, this function representing the simulation picture field. Since the lines of the screen S' are oriented parallel to the x axis and are presumed to be uniform in opacity, the optical transmission of the screen may be written as a function of y alone, $T(y)$.

For a convenient though unnecessary approximation, it will be assumed that the increase in image magnification in going from S'' to S may be neglected. This approximation is valid as long as

$$\frac{\Delta v}{v} \ll 1,$$

a condition which will be fulfilled readily in the usual case.

Out-of-Focus Transformation Without Line Screen

Before evaluating $E(x, y)$ in terms of $E_0(x, y)$ for the actual system, the simple out-of-focus transformation omitting the screen S' will be treated.

Referring to Fig. 3, any point x, y in the sharp-focus plane S'' gives rise to a uniformly illuminated confusion area, A' on S. The boundary of the divergent cone of light from x, y is indicated by the solid lines. If the boundary of A' is symmetrical about an origin taken at x, y then it may be seen that the point x, y on $\underline{S}$ receives light from that portion of S'' which is identical

[4] Actually the light distribution in A' is uniform only in the absence of the line screen S'.

in shape, size, and orientation with A'. The envelope cone of the rays from S'' reaching x, y of S is indicated by the dotted lines.

Thus a measure of the resultant illumination intensity at a point x, y of S may be determined by summing up the contributions from S'' over a region A' of S'' centered at x, y. Since the contributions of points on S'' are weighted according to the function $E_0(x, y)$, the illumination $E(x, y)$ falling on S may be expressed as follows (neglecting constant factors):

$$E(x, y) = \iint_{A'} E_0(x + \xi, y + \eta) d\xi d\eta, \qquad (1)$$

where ξ and η are integration variables measured on an auxiliary set of axes centered[5] at x, y.

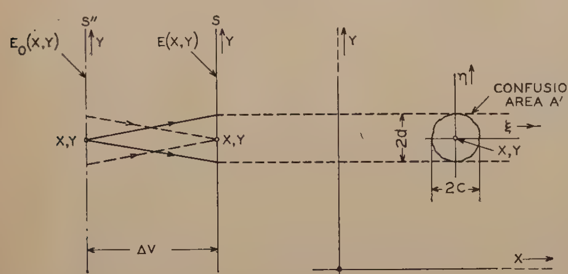

Fig. 3—Analysis of out-of-focus image transformation, without line screen.

In order to interpret (1), it is necessary to adopt some functional representation for $E_0(x, y)$. The form chosen here is the two-dimensional Fourier series used by Mertz and Gray.[6] The light distribution in the plane S'' is set up as an image field as indicated in Fig. 4. The origin of axes is taken at the center of the field as previously described, and the field height and width are $2b$ and $2a$ respectively. Then the intensity of illumination, $E_0(x, y)$, may be represented at any point of the field S'' by the expression

$$E_0(x, y) = \sum_{m=0}^{\infty} \sum_{n=-\infty}^{\infty} a_{mn} \cos \left[\pi \left(\frac{mx}{a} + \frac{ny}{b} \right) + \phi_{mn} \right], \qquad (2)$$

where any component m, n of the summation represents a sinusoidal variation of illumination intensity extending across the image field. The wavelength of the intensity variation is given by

$$\lambda_{mn} = \frac{1}{\sqrt{\dfrac{m^2}{4a^2} + \dfrac{n^2}{4b^2}}},$$

while the orientation of the wave front is given by

$$\tan \theta = \frac{-mb}{na},$$

θ being the angle formed with the x axis.

For mathematical simplicity, (2) may be written in exponential form as follows:

$$E_0(x, y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} A_{mn} e^{i\pi(mx/a + ny/b)}, \qquad (3)$$

where

$$A_{mn} = \frac{a_{mn}}{2} e^{i\phi_{mn}}$$

and

$$A_{-m,-n} = \frac{a_{mn}}{2} e^{-i\phi_{mn}}.$$

Substituting (3) in (1), there results

$$E(x, y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} Y(m, n) A_{mn} e^{i\pi(mx/a + ny/b)}, \qquad (4)$$

where

$$Y(m, n) = \iint_{A'} e^{i\pi(m\xi/a + n\eta/b)} d\xi d\eta. \qquad (5)$$

Thus the result of the simple out-of-focus transformation in the absence of the line screen is to multiply each component m, n by an amplitude or "admittance" factor

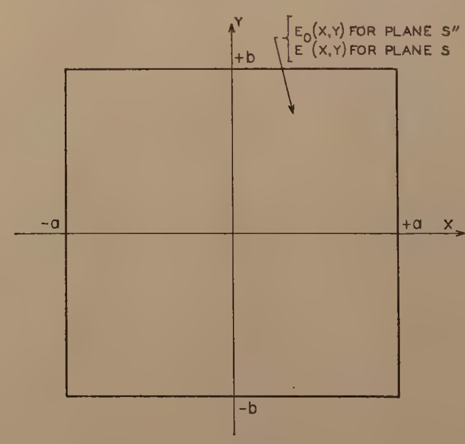

Fig. 4—Image field.

$Y(m, n)$. In general this admittance factor as given by (5) has a decreasing amplitude with decreasing component wavelength, thereby acting as a kind of low-pass filter. For example, if A' is taken to be a rectangle of height $2d$ and width $2c$, then (5) readily yields, neglecting constant factors,

$$Y(m, n) = \left(\frac{\sin \dfrac{\pi mc}{a}}{\dfrac{\pi mc}{a}} \right) \left(\frac{\sin \dfrac{\pi nd}{b}}{\dfrac{\pi nd}{b}} \right). \qquad (6)$$

[5] If a variable-density film is used as the projector aperture A, rather than a shaped hole, then an aperture transmission factor $T(\xi, \eta)$ must be introduced in the integrand of (1).

[6] P. Mertz and F. Gray, "A theory of scanning and its relation to the characteristics of the transmitted signal in telephotography and television," *Bell. Sys. Tech. Jour.*, vol. 13, pp. 464–515; July, 1934.

A plot of (6) for $m=0$, and $b/d=0.707N$ is given by curve A of Fig. 6. Only positive values of n are plotted, since $Y(0, n)=Y(0, -n)$.

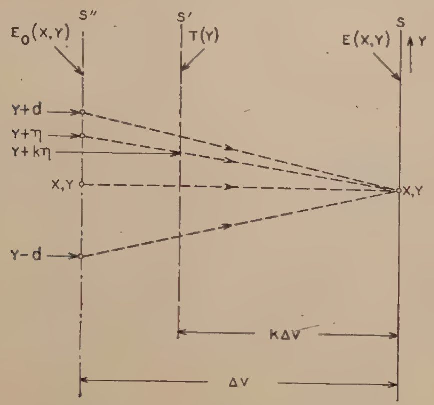

Fig. 5—Comparison of television and simulation systems.

Out-Of-Focus Transformation With Line Screen

Returning to the complete system, Fig. 2, including the line screen S', the manner in which points on S'' contribute to the illumination at the point x, y of S is indicated in Fig. 5. As before, the envelope of the cone of rays from S'' reaching x, y of S is indicated by the dotted lines drawn from the points marked $y+d$ and $y-d$, respectively. Letting $y+\eta$ represent the ordinate of any point of S'' within this cone, it may be seen that the contribution of the point $x+\xi$, $y+\eta$ to the illumination intensity at x, y of S depends not only upon $E_0(x, y)$ as before, but also upon the transmission of the screen S' at the point where the ray from $x+\xi$, $y+\eta$ intersects it. The ordinate of this point of intersection is seen to be $y+k\eta$, where k is the ratio of the $S'S$ spacing to the $S''S$ spacing. Therefore, summing contributions over the region A',

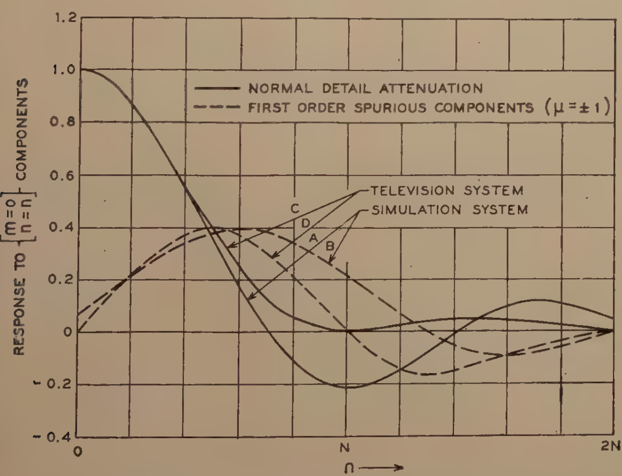

Fig. 6—Analysis of out-of-focus image transformation, with line screen.

$$E(x, y) = \iint_{A'} E_0(x+\xi, y+\eta)T(y+k\eta)d\xi d\eta, \quad (7)$$

where $T(y)$ is the transmission of the screen S'. If the height of the image field $2b$ is taken to contain exactly N black lines of the screen S', N being an integer; then the transmission characteristic $T(y)$ may be written as a single dimensional Fourier series of the form

$$T(y) = \sum_{\mu=-\infty}^{\infty} T_\mu e^{i\pi\mu N y/b}. \quad (8)$$

For simplicity the screen S' will be positioned symmetrically with respect to the x axis. Then the coefficients T_μ will be pure real quantities and

$$T_\mu = T_{-\mu}.$$

Substituting (3) and (8) into (7), there results

$$E(x, y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \sum_{\mu=-\infty}^{\infty} Y(m, n+k\mu N) \cdot T_\mu A_{mn} e^{i\pi(mx/a+(n+\mu N)y/b)} \quad (9)$$

where $Y(m, n+k\mu N)$ is an admittance factor whose form is obtained by substituting $(n+k\mu N)$ for n in (5).

Equation (9) may be rewritten as follows, neglecting a constant factor:

$$E(x, y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} Y(m, n) A_{mn} e^{i\pi(mx/a+ny/b)} + \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \sum_{\mu=-\infty}^{\infty} Y(m, n+k\mu N) \cdot \left(\frac{T_\mu}{T_0}\right) A_{mn} e^{i\pi(mx/a+(n+\mu N)y/b)}, \quad \mu \neq 0, \quad (10)$$

where T_0 is the value of T_μ for $\mu=0$.

The first term in the right member of (10) is identical with the expression given by (4) for the out-of-focus transformation in the absence of the line screen. This term may be called the "normal detail" field, since it contains only components which were present in the original subject matter, attenuated by the admittance factor $Y(m, n)$.

The second term of (10), contributed by the line screen, evidently may be classified as a "spurious detail" field, since the indexes of the components do not correspond to those of the original image.[7]

The significance of (10) may be brought out more clearly by the fact that a component m, n of the original image field, having an amplitude A_{mn}, gives rise in the simulation image, not only to an identical component m, n, having an amplitude $Y(m, n)A_{mn}$; but also to a

[7] However, it is possible for certain components of the spurious field to coincide in wavelength and orientation with normal-detail components.

set of spurious components m, $n+\mu N$, having amplitudes $Y(m, n+k\mu N)$ (T_μ/T_0) A_{mn}. Here μ is any positive or negative integer. If the series of $T(y)$ is rapidly convergent, as it will be if the lines and spaces of the screen are roughly equal in width, and if the transition from "line" to "space" is gradual rather than sharp,[8] then

$$\frac{T_\mu}{T_0} \ll 1, \quad \text{for} \quad |\mu| > 1.$$

Under these conditions, the higher order components may be neglected, and the spurious components derived from a subject component m, n may be written as m, $n \pm N$, having amplitudes $Y(m, n \pm kN)$ (T_1/T_0) A_{mn}. These expressions will be found to be similar to those of the television system.

IV. Television-Image Analysis

A two-dimensional Fourier analysis of the picture degradation imposed by television systems has been widely published.[6] The results of this analysis may be summarized as follows.

Assuming negligible frequency-channel limitation, the television image may be expressed by

$$E(x, y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \sum_{\mu=-\infty}^{\infty} Y_1(m, n) Y_2(m-\mu, n+\mu N) \cdot A_{mn} e^{i\pi(m-\mu)x/a+(n+\mu N)y/b)} \tag{11}$$

where Y_1 and Y_2 are the admittance characteristics corresponding to the transmitting and the receiving scanning apertures (or beams) respectively, and N is the number of scanning lines. If the optical confusion area A' of (5) is replaced by the aperture area of the transmitter or receiver, then $Y_1(m, n)$ or $Y_2(m, n)$ may be found[9] from (5).

As in the case of (9), (11) may be written as the sum of a normal and a spurious field. Thus,

$$\begin{aligned} E(x, y) = &\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} Y_1(m, n) Y_2(m, n) A_{mn} e^{i\pi(mx/a+ny/b)} \\ &+ \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \sum_{\mu=-\infty}^{\infty} Y_1(m, n) Y_2(m-\mu, n+\mu N) \\ &\cdot A_{mn} e^{i\pi(m-\mu)x/a+(n+\mu N)y/b)}, \quad \mu \neq 0. \end{aligned} \tag{12}$$

The first term of (12) is the normal field, being the same as the normal field of (10) if $Y(m, n)$ is replaced by $Y_1(m, n)$, $Y_2(m, n)$. The second term of (12) is the spurious field, there appearing to be somewhat more difference between this term and the spurious field of (10) than was found between the two normal fields. However, the difference is not important, as will be seen.

[8] The line screens used in this work were made by photographic copying of an original ruled screen. The sharpness of line-to-space transition, as well as the relative widths of line and space, may be controlled readily with this method.

[9] This assumes the transmission of the television apertures to be uniform within the aperture boundaries. Otherwise a factor $T(\xi, \eta)$ must be inserted in the integrand of (5), as was pointed out in footnote reference 5 for the optical system.

V. Comparison Between Simulation and Television Formulas

Normal-Field Comparison

Returning to the consideration of the normal fields, definite constants for the television and simulation systems will be chosen to achieve concreteness.

The television apertures will be taken as identical squares of side equal to the scanning-line pitch. The optical area of confusion A' will be taken as a square of side k_1 times that of either television aperture. Also the number of black lines in the line screen will be taken equal to the number of television scanning lines.

Under these conditions we may write, from (6), the normal-field admittance factor for the television system as[10]

$$\begin{aligned} Y_1(m, n) Y_2(m, n) &= Y_1^2(m, n) \\ &= \left[\frac{\sin\frac{\pi m}{N}}{\frac{\pi m}{N}}\right]^2 \left[\frac{\sin\frac{\pi n}{N}}{\frac{\pi n}{N}}\right]^2 \\ &= Y_1^2(m, 0) \cdot Y_1^2(0, n), \end{aligned} \tag{13}$$

and the corresponding quantity for the simulation system as,

$$\begin{aligned} Y(m, n) = Y_1(k_1 m, k_1 n) &= \left[\frac{\sin\frac{\pi k_1 m}{N}}{\frac{\pi k_1 m}{N}}\right] \left[\frac{\sin\frac{\pi k_1 n}{N}}{\frac{\pi k_1 n}{N}}\right] \\ &= Y_1(k_1 m, 0) Y_1(0, k_1 n). \end{aligned} \tag{14}$$

Expanding $Y_1^2(m, 0)$ in a Taylor's series,

$$Y_1^2(m, 0) = 1 - \frac{1}{3}\left(\frac{\pi m}{N}\right)^2 + \frac{2}{45}\left(\frac{\pi m}{N}\right)^4 - \cdots. \tag{15}$$

Similarly,

$$Y_1(k_1 m, 0) = 1 - \frac{k_1^2}{6}\left(\frac{\pi m}{N}\right)^2 + \frac{k_1^4}{120}\left(\frac{\pi m}{N}\right)^4 - \cdots. \tag{16}$$

Equating coefficients of $(\pi m/N)^2$ in (15) and (16), we find $k_1 = \sqrt{2}$.

Substituting this value for k_1 in (16), we have

$$Y_1(k_1 m, 0) = 1 - \frac{1}{3}\left(\frac{\pi m}{N}\right)^2 + \frac{1}{30}\left(\frac{\pi m}{N}\right)^4 - \cdots$$

[10] Setting $a=b$, which incurs no essential loss of generality.

which is a close approximation to (15) for the range of m which is of interest. The quality of the approximation is shown by the degree to which curve A matches curve C of Fig. 6. Thus we may write

$$Y_1(k_1m, 0) \simeq Y_1^2(m, 0), \quad k_1 = \sqrt{2}.$$

Similarly,

$$Y_1(0, k_1n) \simeq Y_1^2(0, n), \quad k_1 = \sqrt{2}.$$

Therefore, for the constants chosen, we may write

$$Y(m, n) \simeq Y_1(m, n)Y_2(m, n),$$

and the normal field expressions for the simulation system as given by (10), and the television system as given by (12) agree.

Spurious-Field Comparison

Continuing with the same constants as in the preceding discussion, we proceed to an evaluation of the television-image spurious field as given by the second term of (12). The meaning of this term may be expressed in the fact that an original subject component m, n of amplitude A_{mn}, gives rise to spurious components $m-\mu$, $n+\mu N$ at amplitudes $Y_1(m, n)Y_2(m-\mu, n+\mu N)A_{mn}$, μ being any positive or negative integer. The amplitude expression decreases rapidly with increasing absolute magnitude of μ. Thus the consideration of spurious components may be confined to those for which $|\mu| = 1$. Accordingly, we may neglect μ compared to m with negligible error.

The resulting spurious-component expression for the television image is m, $n \pm N$, at an amplitude of $Y_1(m, n)Y_2(m, n\pm N)A_{mn}$. This expression gives the same spurious-component indices as were found for the simulation system, so all that remains is to compare the corresponding amplitude expressions of both systems.

Omitting A_{mn} as being common to both amplitude expressions, the spurious amplitude factor for the television system with square apertures may be written as

$$Y_1(m, n)Y_2(m, n \pm N) = Y_1^2(m, 0)Y_1(0, n)Y_1(0, n \pm N).$$

The corresponding quantity for the simulation system is

$$Y(m, n \pm kN)\frac{T_1}{T_0} = Y(m, 0)Y(0, n \pm kN)\frac{T_1}{T_0}.$$

It was shown in the previous section that

$$Y(m, 0) = Y_1(k_1m, 0) \simeq Y_1^2(m, 0), \quad k_1 = \sqrt{2}$$

so that it is sufficient to compare

$$Y_1(0, n)Y_1(0, n \pm N) \quad \text{with} \quad Y(0, n \pm kN)\frac{T_1}{T_0}.$$

There are two elements of control in the latter expression which may be used to obtain agreement of these expressions. First, variation of the factor k (which may be remembered as determined by the positioning of the line screen) permits shifting of the expression along the axis of n. Second, the magnitude of the expression may be controlled by the factor T_1/T_0, which is determined by the nature of opacity variation across the lines and spaces of the line screen.[11]

Since the two expressions are fairly similar, these two adjustments permit a reasonably good agreement. A comparison is shown in Fig. 6, $Y_1(0, n)Y_1(0, n-N)$ being given[12] by curve D and $Y(0, n-kN)\ T_1/T_0$ (for the values $k=0.6$, $T_1/T_0=0.4$, and $k_1=\sqrt{2}$) by curve B. The divergence of the two curves for values of n above $0.7N$ is not important because of the small amplitudes of A_{mn} found in this region. The fact that the simulation curve B does not pass through the origin, for the choice of $k=0.6$, means that there will be some structure showing even when the original subject matter is a flat field. However, this effect is small.

Thus we find that there is a substantial agreement between the simulation and the television system, both as to normal and spurious components, for the square apertures assumed. A similar analysis holds for other types of apertures. If the transmitter and receiver apertures are alike in configuration and size, then the optical aperture should preferably be chosen to be of the same configuration,[13] and the focusing adjusted so that the optical confusion area is $\sqrt{2}$ (in linear dimensions) times that of either television aperture. The number of black lines in the line screen should always be chosen equal to the number of active scanning lines in the television system. Finally, the parameters k and T_1/T_0 should be chosen to obtain a match between the curves B and D of Fig. 6; that is, between the spurious-component amplitude factors of the optical and television systems for original subject components lying parallel to the scanning lines ($m=0$).

Frequency-Channel Limitation

The foregoing has neglected the effect of a finite frequency bandwidth in the television transmission channel. That an aperture characteristic may be used to simulate an electrical low-pass filter is well known, although the simulation has its limitations where the effect of a sharp cutoff is desired. Following the usual practice in this matter, it will be assumed that the actual electrical-filter characteristic may be replaced by an equivalent one having a gradual cutoff, which in turn may be simulated by an aperture characteristic. Then,

[11] A convenient method of controlling the T_1/T_0 ratio is to adjust, via photographic processing, the ratio of optical transmission of the "lines" and "spaces."

[12] Because of symmetry, only one sign of the $\pm$ need be considered.

[13] The more important consideration is that the optical aperture should have the same admittance in any direction as the television apertures. For instance, the effect of a scanning beam having non-uniform transmission over its area may be closely simulated by the shaped-hole type of aperture.

Fig. 7—Resolution diagram, with spurious structure.

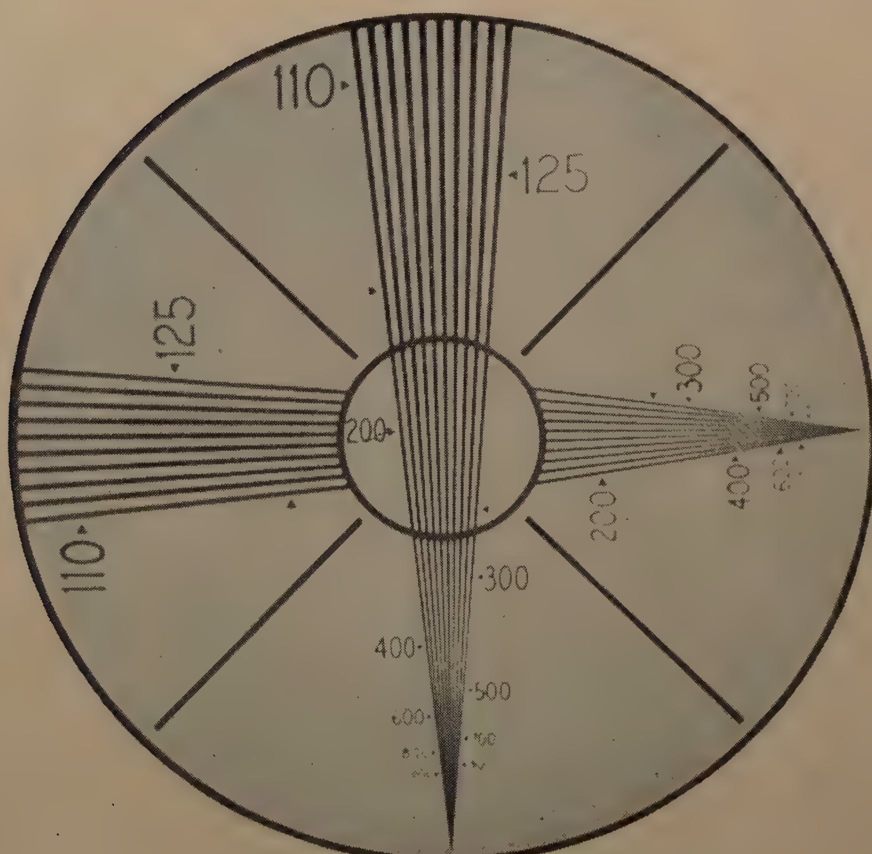

Fig. 8—Resolution diagram, without spurious structure.

since the principal effect of electrical frequency attenuation is upon the components of the image perpendicular to the scanning, the frequency limitation may be taken into account by increasing the horizontal dimension (scanning assumed horizontal) of the confusion area of the simulation system over the value arrived at from previous considerations.

This is exemplified in the square-aperture television system by the following procedure. The effect of the filter cutoff in the direction of scanning is evaluated as that due to a rectangular aperture of length (in scanning direction) $2c'$. Then if $2c$ is the length of the transmitter and receiver apertures, the required horizontal dimension of the optical simulation aperture is

$$2c'' = \sqrt{(2c)^2 + (2c)^2 + (2c')^2}.$$

VI. Discussion and Results

Photographs of simulated television pictures obtained with the apparatus of Fig. 1 are shown in Figs. 7, 9, 11, 13, and 14. In each of these pictures the scanning apertures of the equivalent television system have been taken to be squares of side equal to the scanning-line pitch. No allowance for electrical filtering has been made, the aperture attenuation being considered as the cutoff of the television channel.[14] The other parameters

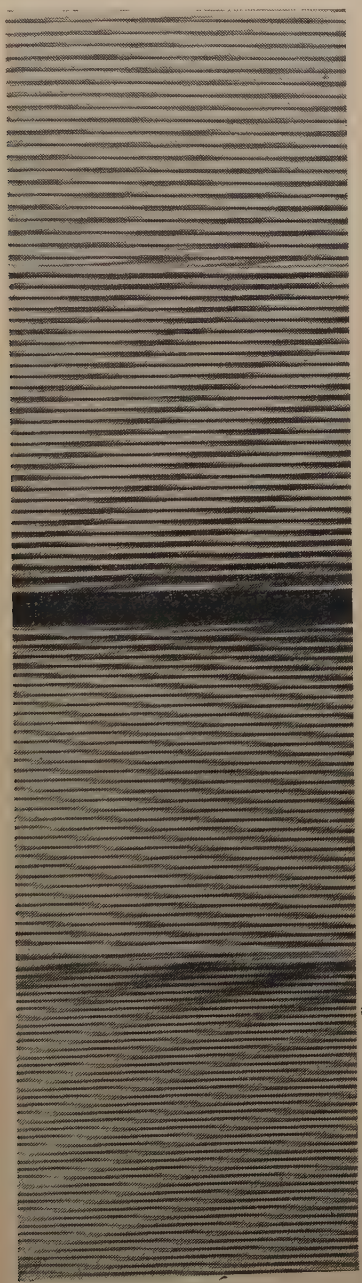

Fig. 9—Parallel lines, with spurious structure.

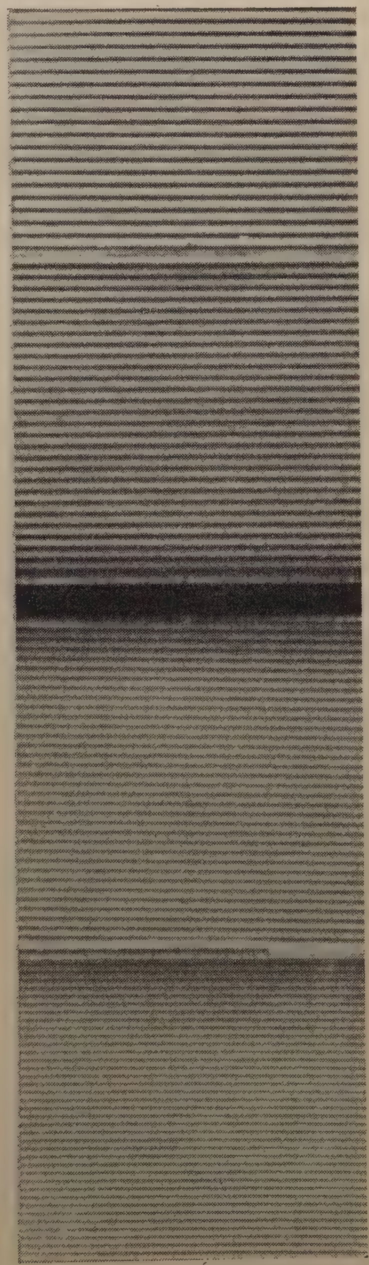

Fig. 10—Parallel lines, without spurious structure.

[14] Within the limitations previously pointed out, the inclusion of electrical frequency-attenuation effects would have been a simple matter. However, because of the unavoidable resolution loss in reproducing the photographs, further refinements were considered unnecessary.

are approximately those given previously for curves A and B of Fig. 6.

Fig. 7 shows a simulation picture of a resolution diagram, there being the equivalent of approximately 360 scanning lines across the height of the picture. The effect of the spurious components, as given by the second term of (10), may be seen in the steps along the lines of the horizontal wedge, in the spurious pattern in the right half of this wedge, and in the serrations appearing on the numbers. It may be observed that the resultant resolution of lines in the horizontal wedge is markedly lower than that for the vertical wedge.

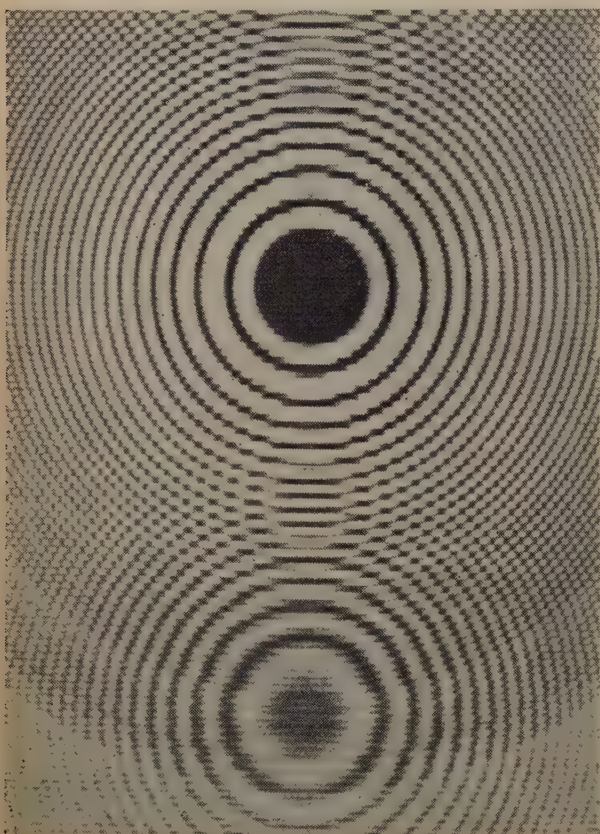

Fig. 11—Zone plate, with spurious structure.

An interesting comparison may be made between Fig. 7 and Fig. 8. Fig. 8 is a simulation picture, purporting to represent the same television picture as Fig. 7, made by using an out-of-focus optical system omitting the scanning structure. In order to take account of this omission of spurious components, the height of the figure of confusion has been increased by a factor of 1.4, an average of the factors suggested by various investigators[15] to take account of the vertical-resolution degradation due to scanning structure. It may be seen that a completely satisfactory equivalence factor[16] expressing the effect of the spurious components in terms of resolution degradation may be difficult to arrive at in general.

Fig. 9 is a simulation photograph of four sections of parallel lines, each having a different pitch, arranged so that the subject lines form a small angle with the scanning lines. There are approximately 400 lines contained in the picture height. Fig. 13 is the corresponding simulation picture omitting scanning structure and having a 1.4-times-greater vertical aperture.

Fig. 11 is a particularly interesting illustration of the peculiar effects arising from the spurious component field. The original subject matter was the zone plate of

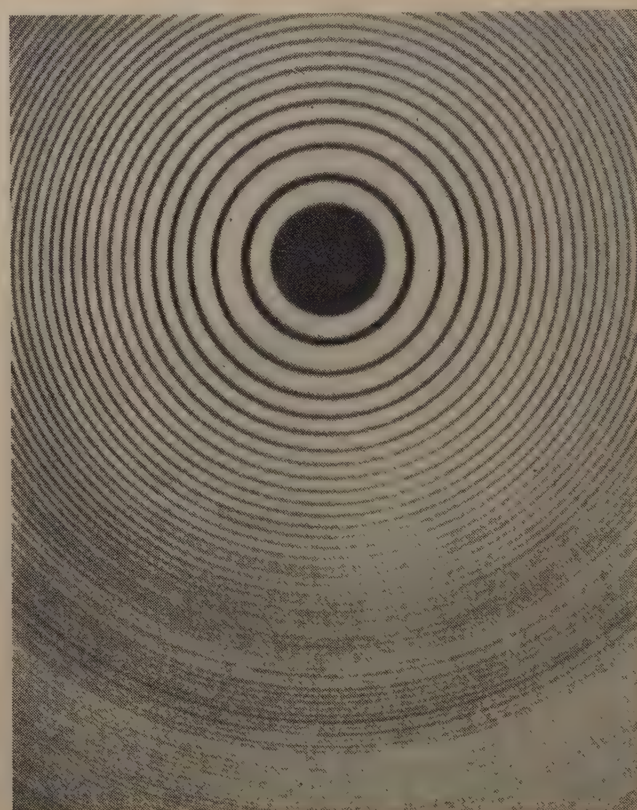

Fig. 12—Zone plate, without spurious structure.

Fig. 12, the spurious structure giving the effect of a ghost image or echo of the original.[17] Beats between this spurious image and the original may also be observed. The corresponding nonstructure simulation picture is shown in Fig. 12.

Figs. 10 and 11 are simulation pictures[18] of 441-line television pictures which have about 15 per cent vertical blanking. The scanning structure shows up throughout the outdoor scene, and is evident in the portrait of the girl, around the eyes, teeth, and hat.

Fig. 15 is an out-of-focus picture of the zone plate of Fig. 12, which illustrates the significance of aperture

[15] See footnote reference 3. Baldwin lists the values which have been obtained for this factor.

[16] The estimates of this factor were usually based on the use of general subject matter, rather than the specialized type of subject used above.

[17] A picture similar to Fig. 11, showing a zone plate after transmission over a telephotograph system is given by Mertz and Gray. (See footnote reference 6.)

[18] Scene of Fig. 13 reproduced by courtesy of Loucks and Norling. Scene of Fig. 14 reproduced by courtesy of Fox Movietone News.

Fig. 13—Simulation picture of 441-line television picture.

Fig. 14—Simulation picture of 441-line television picture.

admittance characteristics of the type shown in curve *A* of Fig. 6. The confusion area is square and is arrayed with its sides horizontal and vertical. The "waves of sharpness," which may be seen principally along vertical or horizontal radii as the pitch of the rings diminishes, correspond to the lobes of the aperture admittance curve appearing between successive crossings of the zero axis. The first crossover point of the aperture admittance characteristic was found very useful in checking experimentally the size of confusion area. That practically no lobular resolution is visible along 45-degree radii is due to the fact that the aperture appears as diamond-shaped along these directions, the lobes of a diamond aperture admittance characteristic being very small compared to those for a square or rectangle.

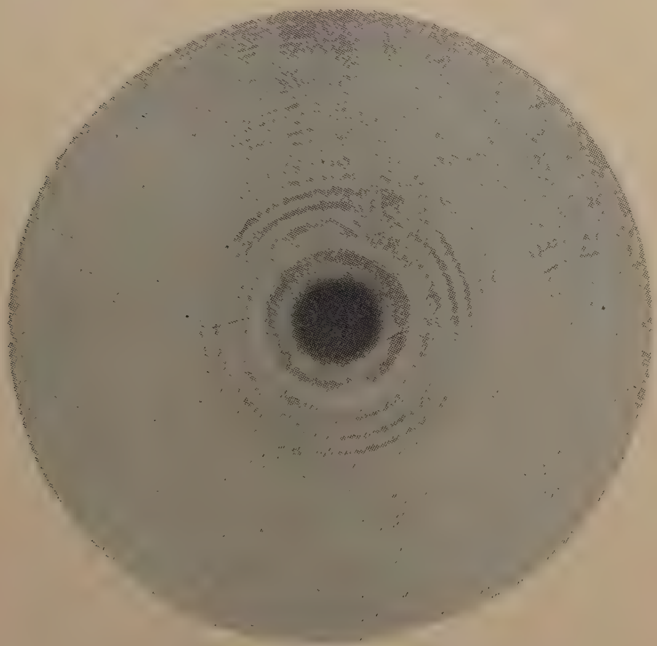

Fig. 15—Zone plate; illustration of aperture effect.

There are several alternative optical arrangements which may be used for producing simulation images similar to those discussed above. One variation, which permits the projection of large pictures viewed on a reflecting screen, is obtained by placing the line screen behind the projector lens close to the film plane. Another variation, of interest because it produces an almost exact simulation, may be obtained by using a line screen in conjunction with two out-of-focus transformations in tandem. However, this would be suitable only for photographic work, due to the low picture brightness obtained with this arrangement.

Simulations of both 240- and 441-line television pictures, still and motion, have been set up, and on numerous occasions compared with the images produced by the corresponding television systems. The fundamental similarity between the simulation and television images was strikingly apparent even at close viewing distances; and the quantitative comparison agreed with that predicted by theory. Simulations of 525- and 625-line television images have also been made. The conversion of the simulation picture from one grade of television image to another was very simple, requiring only a change of line screens and readjustment of the focusing dial.

VII. Appendix

Diffraction Calculation of an Out-of-Focus Optical System for a Rectangular Aperture

The geometrical theory which has been used in the paper to describe the figure of confusion of the optical system is subject to appreciable error for planes near the sharp-focus plane. In order to investigate the magnitude

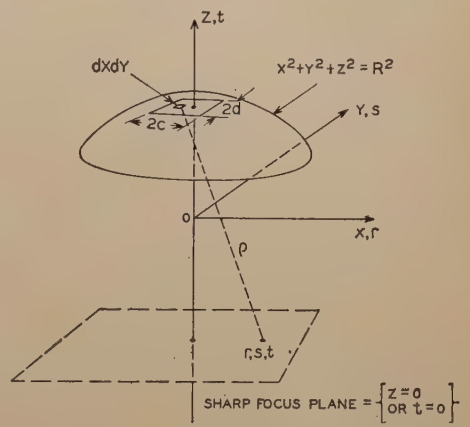

Fig. 16—Diffraction calculation of the confusion area for an out-of-focus optical system.

of the error, a diffraction calculation of the system has been made. The line screen is omitted from the calculation, since a simple estimate shows diffraction effects arising from this source to be small for the dimensions used.

The formulation of the diffraction solution follows Fig. 16. It is assumed that the wave front of the light emerging from the lens, due to a point source located on the optical axis at the film plane, is a section of a sphere with center on the optical axis at the sharp-focus plane. The radius R of the sphere is the image distance of the optical system. The active portion of the spherical wave front is that section bounded by the rectangular aperture of sides $2c$ and $2d$ oriented parallel to the x and y axes, respectively. Then, neglecting the varying inclination of surface elements of the sphere over the aperture boundary, the light amplitude at any point of the image space r, s, t may be written as

$$a = \iint_{\text{aperture}} \sin 2\pi\left(\frac{\tau}{T} - \frac{\rho}{\lambda}\right) dxdy,$$

where ρ = the distance from a surface element $dxdy$ to the point r, s, t,

τ = time

λ = wavelength of light (assumed monochromatic).

By assuming the maximum values of the variables x, y, r, s, t to be small compared to R, the following result is obtained for the light intensity at a point r, s, t. Neglecting constant factors, we find,

$$J = ([C(a_1 + a_2r) + C(a_1 - a_2r)]^2 + [S(a_1 + a_2r) + S(a_1 - a_2r)]^2) \cdot ([C(a_3 + a_2s) + C(a_3 - a_2s)]^2 + [S(a_3 + a_2s) + S(a_3 - a_2s)]^2) \quad (17)$$

where $C(\ )$ and $S(\ )$ are the Fresnel integrals, being given by

$$C(x) = \int_0^x \cos\frac{\pi}{2} v^2 dv,$$

$$S(x) = \int_0^x \sin\frac{\pi}{2} v^2 dv,$$

and where, for positive values of t,

$$a_1 = c\sqrt{\frac{2t}{\lambda R(R-t)}}$$

$$a_2 = \sqrt{\frac{2R}{\lambda t(R-t)}}$$

$$a_3 = a_1 \frac{d}{c}.$$

For negative values of t, the absolute magnitude of t must be used in (17) and in the following formulas.

$$a_1 = c\sqrt{\frac{2t}{\lambda R(R+t)}}$$

$$a_2 = \sqrt{\frac{2R}{\lambda t(R+t)}}$$

$$a_3 = a_1 \frac{d}{c}.$$

Fig. 17 shows a plot of J in the plane $t = -2$ inches, for $s = 0$ and the representative values $R = 76$ inches, $c = d = 1/2$ inch, $\lambda = 5000$ angstrom units. The aperture admittance corresponding to this intensity distribution is shown in Fig. 18, along with the admittance curve obtained from geometrical considerations. There is seen to be very good agreement between the two, so that the

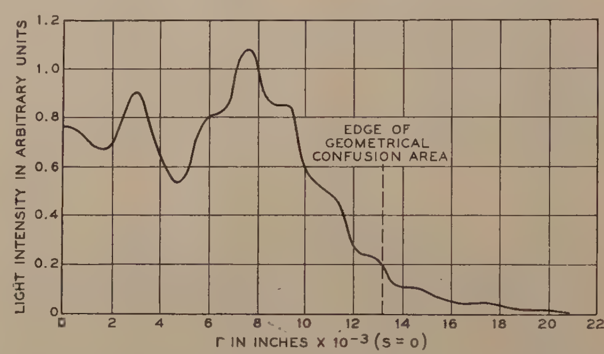

Fig. 17—Diffraction calculation. Light distribution in the out-of-focus plane $t = -2$ produced by an axial point source in the film plane.
$R = 76$ inches
$c = d = \frac{1}{2}$ inch

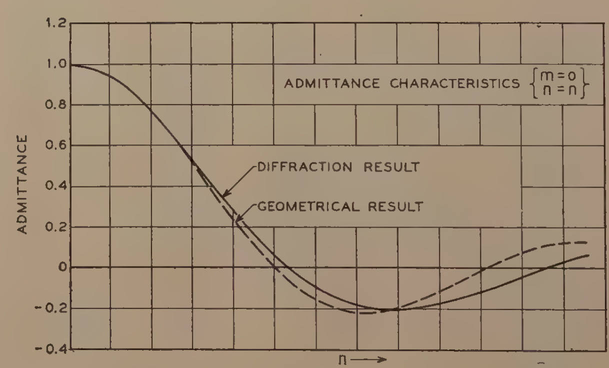

Fig. 18—Comparison of geometrical and diffraction admittance calculations. Solid curve calculated from curve of Fig. 17.

geometrical theory may be used for the given dimensions without appreciable error. For values of t less than one inch, however, the error becomes noticeable, and should be considered.

Problems in the Manufacture of Ultra-High-Frequency Solid-Dielectric Cable*

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Summary—This paper deals with the various types of semiflexible solid-dielectric transmission lines suitable for use at ultra-high frequencies. Five general types are described, and their applications discussed. These are (a) coaxial, (b) dual, (c) dual-coaxial, (d) low-capacitance, and (e) high-impedance. Types (a) and (b) are general-purpose lines, while type (c) is used in direction-finding equipments, instrument-landing systems, etc. High-impedance cables find use in cathode-follower and special delaying circuits.

The manufacture of these transmission lines is discussed, and the various problems associated with extruding and braiding examined. The practical question of penetration of the braid into the dielectric and jacket is considered, and the recent steps taken to eliminate this problem discussed.

The contribution of the individual components of the cable to the total loss is examined, and an apparatus described which enables a direct measurement of braid resistance to be made at 150 megacycles. A typical example is given for RG-8/U cable, showing close agreement between the measured total loss of the line and the loss obtained by a summation of the individual losses of dielectric, inner conductor, and braid. The effect of varying the braid construction is demonstrated. The electrical testing of high-frequency transmission lines poses some problems, and a brief description, limited for security reasons, of the various tests and equipments is given. The author concludes by recommending immediate consideration of standardization problems in this field.

Introduction

THE VERY rapid advances made in the electronic field in the last decade, and particularly those necessitated by the exigencies of war, have brought many new and interesting developments to fruition. For obvious reasons, many of these developments must remain closely guarded secrets until after this period of international conflict, but some of them can now be discussed in general terms, and the progress made outlined. In this paper it is proposed to deal briefly with a not very spectacular, but nevertheless highly important, component of modern electronic equipments, the ultra-high-frequency solid-dielectric cable. Such cables have now attained a noteworthy place in the components list, and are destined to play a more important role in the future.

The Nature of Solid-Dielectric Lines

One of the principal factors necessitating the development of solid-dielectric semiflexible cables was the design of electronic equipments for moving vehicles such as tanks, airplanes, and trucks; and for demountable equipments such as instrument-landing systems, direction finders, gun-directing devices, etc. These equipments called for cable components that would be light, portable, easily assembled and disassembled, semiflexible, and having few (if any) maintenance problems. Not the least of the requirements, however, was the ability to obtain such cables in large quantities with the minimum of expansion of production facilities.

The standard rigid line, consisting essentially of a center conductor coaxially placed in a solid-copper tube and supported by rigid discs, washers, or spacers of dielectric material uniformly spaced, is well known. To obtain satisfactory operation of such lines, it is necessary to use somewhat complicated and expensive "plumbing" methods to join the lines to equipments and themselves, and to install a special nitrogen or inert-gas atmosphere under pressure to prevent "breathing" of the line and consequent moisture deposition inside the structure. Such an installation is costly, but, where the assembly is permanent, affords highly satisfactory operation, since it can be rigidly installed and with periodical maintenance check-ups can be expected to maintain its original qualities. The mechanical construction of these lines enables a high degree of uniformity of electrical characteristics to be obtained, and because of their rigid construction from solid conductors, the exact value of impedance and attenuation can be calculated before the line is even constructed. With semiflexible ultra-high-frequency transmission lines, however, the maintenence of uniform electrical characteristics is much more difficult, particularly when it is remembered that such lines are expected to operate over a wide temperature range, −40 degrees centigrade to +80 degrees centigrade, to be capable of much abuse in the field, and to be adaptable for the many different types of equipment now in use.

At the present time, semiflexible cables for ultra-high-frequency use fall into the following general types: coaxial, dual, dual-coaxial, low-capacitance, and high-impedance.

In the case of coaxial types, we have a center conductor insulated with a synthetic dielectric, a braid which acts as the return conductor and also as an electromagnetic shield, and an outer protective sheath of synthetic resin. In certain cases, especially for shipboard use, where installation in conduit is a requisite, a galvanized steel or aluminum armor is put over the sheath, which affords both mechanical protection and additional electrical shielding. The dual types are similar to the coaxial types, except that the two inner conductors are imbedded in the primary insulation. The dual-coaxial types of cables are special designs for specific problems, such as that in connecting up the antenna arrays of

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instrument-landing systems, in certain direction-finding equipments, etc.

The low-capacitance cable consists of a center conductor held in position in a thin-walled tube of dielectric by an insulating thread spirally wound around the center conductor. Over the tube is put the usual braid and jacket. Such a design, by virtue of its relatively large volume of air, has a low effective dielectric constant, and therefore the capacitance per foot is also low.

The high-impedance type of cable is a special design for those equipments where an impedance of the order of 1000 ohms is required in the cable. Such a cable comprises an inert supporting core on which is wound a close-lay spiral of enameled wire. Over this is put a synthetic insulation and then the conventional braid and sheath. Fig. 1 shows the various types of cables discussed above.

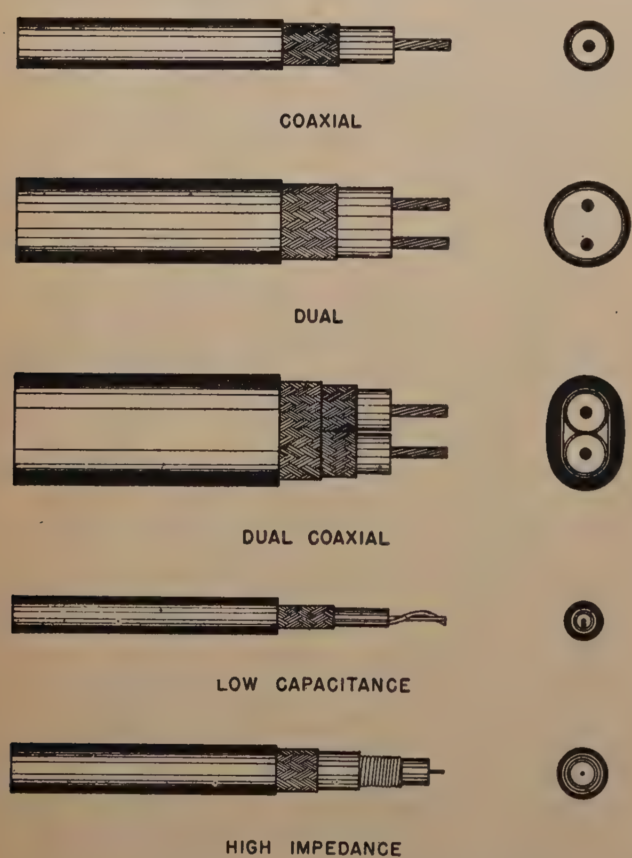

Fig. 1.—General types of ultra-high-frequency cables.

The Manufacture of Solid-Dielectric Cables

The chief operations in the manufacture of cables for use at ultra-high frequencies are those of extrusion and braiding. Extrusion is a process for forcing materials in a semisolid state through a suitable orifice such that a definite shape is obtained. The first operation in cable manufacture is the extruding of the primary insulation on the center conductor, which is done by means of a specially designed plastics extruder. Such an extruder consists essentially of a heated cylinder through which the insulating material is forced by means of a rotating screw, and in which the material is brought to the right state of plasticity, a breaker-plate assembly which serves to build up pressure and to ensure that all undispersed or foreign matter is removed from the material, a cross head through which the center conductor to be insulated is fed, and a tip and die assembly which forms the insulation around the center conductor to the desired shape. The various parts of the machine can be seen by reference to Fig. 2. Although at first sight it might appear that this process is but little different from that employed in the manufacture of conventional rubber-insulated wires, the fact that we are dealing with plastic materials of higher softening point and different degrees of plasticity, necessitates machine modifications, while the necessity for maintaining a high degree of uniformity introduces manufacturing problems of no mean extent. It is obvious that, for successful service use, certain electrical parameters must be closely controlled, and of these parameters, the characteristic impedance introduces the first problem from the mechanical point of view.

Since it is desired to keep the characteristic impedance of the cable as uniform as possible and at a fixed value, the ratio of the diameter of the inner conductor to the diameter of the outer conductor must be held to a close tolerance. For most applications, it has been decided that, with a nominal impedance of 50 ohms, a tolerance of ± 2 ohms is the maximum permissible. To achieve this, it is necessary to hold the diameter of the dielectric to within ± 15 mils. Since it is necessary during manufacture to have some little leeway in tolerance, it means in practice that to keep rejections to a minimum, even tighter dimensional tolerances must be maintained. When it is recognized that this tolerance must be maintained while the cable is being produced at speeds up to 100 feet per minute, it will be appreciated that the equipments must be functioning correctly, the operators well-trained and supervised, and constant production engineering maintained.

To ensure uniform values of attenuation, and particularly to maintain the lowest values possible, the dielectric material must be rigidly inspected before use, and handled with the greatest of care to avoid contamination. Since the velocity of propagation is a function of the dielectric constant of the insulating material, care must be taken in manufacture to avoid the presence of voids or discontinuities in the dielectric and to see that the material is applied uniformly as regards density.

When the frequency of operation becomes very high, indeed, we find that additional problems are present. At these frequencies, the presence of any form of discontinuity will cause trouble due to reflections and standing waves. This may occur even though the cable is well within the tolerance values for size called for in the specification. To overcome such problems, the nature and extent of which are only just becoming very painfully apparent, the highest degree of skill will be

required from all phases of engineering, the raw-material experts, the mechanical designers, and the electrical engineers.

After the extrusion of the primary insulation, the braid or outer conductor is put on by a process referred to as "braiding." Two main types of braiding machines are in operation, the so-called "Wardwell" type, and the "New England Butt" type. In the Wardwell machine we have two carriages, each having twelve bobbins containing the braid wires, rotating horizontally in opposite directions, the individual bunches of wires from the bobbins being deflected by guides to interweave them in the form of a basket weave. Such a braider is shown in Fig. 3. For large cables, the New England Butt type is often employed; here the bobbins, usually forty-eight in number, are mounted vertically and rotate in a horizontal plane around the periphery of the machine, forming a basket weave around the centrally located cable.

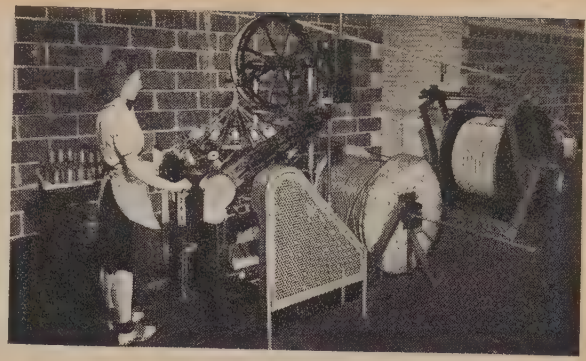

Fig. 3—Copper-wire braiding machine.

with polyethylene, the braid wires may have a tendency to imbed in the dielectric, particularly if the cable has been heated or if the braid has been designed with a relatively short lay and applied with a high tension. It is even more common for the braid wires to be imbedded

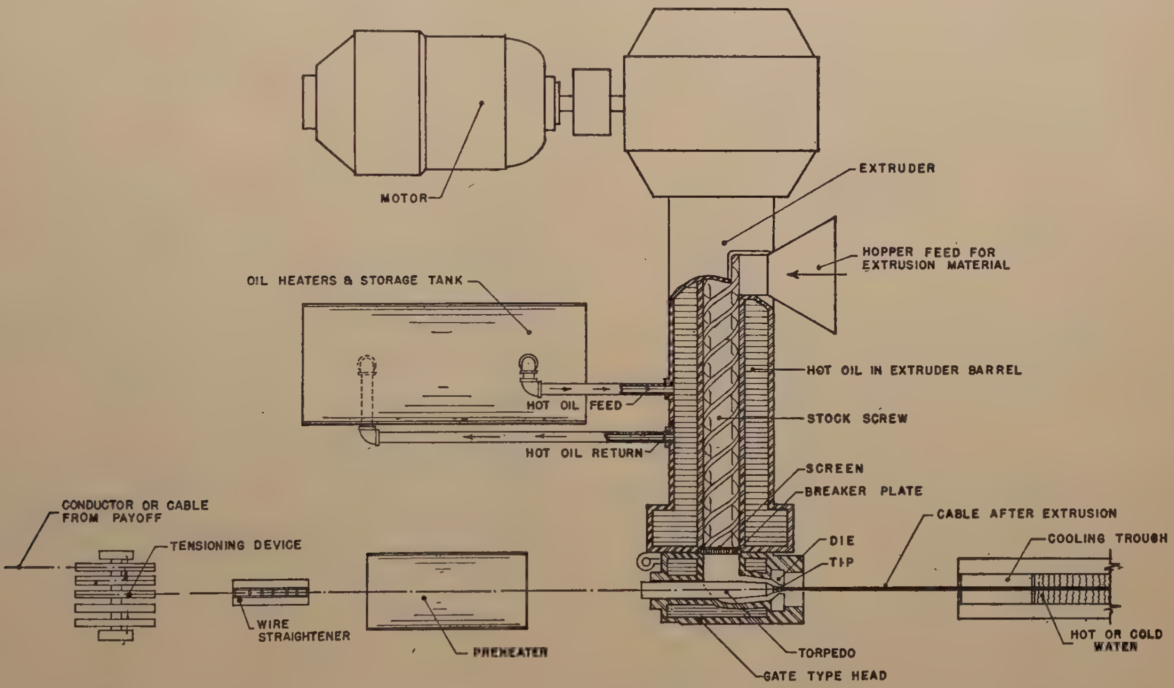

Fig. 2—Schematic cross section of extruder.

In general, apart from design characteristics, the problems in braiding are purely mechanical, and the success of good braiding depends chiefly on the care with which the individual wires are wound on the bobbins for braiding.

There is one problem concerning the braid which is of particular interest to the equipment manufacturer, and that is the adhesion or penetration of the braid to either the primary insulation or the jacket. When softer dielectrics than the polyethylene now standard were employed, it was quite common for the braid wires to be buried so deeply in the dielectric, due to the tensions exerted during braiding, that it was a matter of great difficulty to strip the braid wires back preparatory to connector assembly. It sometimes happens that even in the jacket; this is caused by applying the plastic jacketing material to the braided wire in a very soft state and through a tip-and-die combination causing the hot plastic to be forced through the interstices of the braid. On cooling, the braid wires are often found to be completely imbedded in the jacket. In such cases, it is almost impossible satisfactorily to prepare a cable end for connector assembly. The use of special tubing tips and dies which serve to lay a tube of the protective jacketing material on the braid under the right pressure, has overcome this problem. The presence of jacketing material in the braid also serves to introduce additional attenuation losses in the line and to give it attenuation instability with flexing.

The sheath, or jacket, is applied with the same type

plastic tuber as the primary insulation, and affords neither more nor fewer problems than those discussed under that subject.

Design Components

One of the most important factors affecting the design of a high-frequency cable is the nature of the dielectric material to be employed. Such a dielectric has to have the lowest attainable electrical losses, and these losses must be substantially constant over a wide frequency band and a considerable range of operating temperatures. Apart from these electrical requirements, the dielectric must also be capable of installation at temperatures as low as −40 degrees centigrade and yet support the weight of the center conductor at elevated temperatures of +85 degrees centigrade without flow. The search for suitable materials has been going on for some considerable time, and it cannot yet be said that a completely satisfactory material has been developed. At the present moment in this country and Great Britain, the preferred insulation is polyethylene, a high-molecular-weight hydrocarbon of the paraffin series. The properties of this material are given in Table I. This material is

Table I

Polyethylene, Average Properties

Tensile strength, pounds per square inch	1700
Coefficient of thermal expansion	10.5×10^{-5}
Rockwell hardness	R13
Specific gravity	0.92
Elongation, per cent	500
Heat distortion point, degrees Fahrenheit	140
Water absorption after 24-hour immersion, per cent	0.01
Dielectric strength, volts/mil, 3 to 15 mils thick	1000 to 1500
Volume resistivity, ohm-centimeters	10^{17}
Dielectric constant	
60 cycles	2.25 to 2.27
1000 cycles	2.25 to 2.27
1 megacycle	2.25 to 2.27
1000 megacycles	2.65 to 2.27
Power factor	
60 cycles	0.0002 to 0.0004
1000 cycles	0.0002 to 0.0004
1 megacycle	0.0002 to 0.0004
1000 megacycles	0.0002 to 0.0004

now being manufactured in large quantities, and a high degree of uniformity is obtained. Its low value of power factor (of the order of 0.00030) and its attendant low dielectric constant (approximately 2.27) are maintained over a wide range of frequencies, while its flexibility at low temperatures and its rigidity at elevated temperatures are satisfactory.

By skillful operating technique, a high degree of extrusion precision can be obtained, which enables good control of impedance and other electrical parameters of the final product to be maintained.

The center conductor of a high-frequency solid-dielectric cable is either concentric-lay, stranded, or solid, dependent upon the general considerations applying to its use. Stranded conductors are chiefly employed where the greatest degree of flexibility of the cable is required; solid conductors are generally employed where the lowest electrical losses are necessary and also where a cable is desired to have the highest value of initial corona-starting potential. In order to reduce the losses still further, it is common practice now to use silver-plated conductors. For certain applications, it is desirable to have a cable which has a high loss per unit length, and for these cables, the center conductor usually consists of a nichrome wire. To maintain constancy of impedance, it is necessary for the center conductor to be manufactured with a high degree of uniformity, and some of the troubles of the earlier high-frequency solid-dielectric lines have been traced back to irregularities of stranding and dimensions. The braid consists of a basket weave of wires which vary in number, size, and mode of application, dependent on the size of the cable and the use to which it is to be put. For the smaller size of cables, it is usual to have 24 separate ribbons of braid wires in the construction, each ribbon comprising six to ten individual wires of no. 33 or no. 34 American Wire Gauge. The lay of the braid, or number of ribbons of wires per inch, is determined by the electrical and mechanical properties required. Since at the frequencies employed the current travels on the surface of the wires, it is obvious that the fewer the jumps the current has to make per unit length, the lower the loss. On the other hand, the shorter the lay of the braid, the more flexible is the cable, and the more it is able to withstand repeated flexings without failure or deterioration of electrical characteristics.

It must be confessed that many of the braid designs currently employed were chosen for expediency rather than from a sound engineering design, but constant improvement is being made, and it is to be hoped that we will be able in future years to design cables a little more scientifically than in the past.

Since plain copper wire has a tendency to oxidize in air and also to corrode in the presence of electrolytes such as salt, it is sometimes found advisable to use tinned-copper wires for the braid instead of plain copper wire. The presence of oxidation and/or corrosion products on copper-braid wires not only causes an increase in the electrical loss of the cable, but also gives rise to fluctuating readings on equipments due to contact-resistance variations when the cable is shifted or flexed. To lower the losses, and also to avoid this variation of contact resistance, especially for cables used as test leads in precision testing equipments, it is now quite common to use a silver-plated copper-braid wire.

In certain applications, where good shielding is required, a second braid is sometimes employed. In such double-braided cables it is usual to take advantage of the electrical properties to be obtained by the use of a long-lay braid, by employing as long a lay as practicable for the inner braid, maintaining the outer braid with a short lay for mechanical considerations. Consideration has been given, from time to time, to the use of more than two braids for additional shielding, but it has now been demonstrated that the advantages gained by the employment of a third shield are not sufficient to warrant the additional expense and problems of manufacture and installation. Such shielding problems are best

handled by investigating the over-all shielding of the individual equipments concerned.

The jacket, or sheath, acts primarily as a protection of the cable structure therein. It should be flexible, resistant to abrasion, oil, gasoline, water, hydraulic-brake fluids, etc., and should also preferably be non-inflammable. The most commonly used jacketing materials for high-frequency cables are plasticized vinyl compounds such as vinyl chloride or vinyl chloracetate, sold under the trade names of Vinylite and Geon. It has recently been found that, where the greatest degree of constancy of operation of high-frequency solid-dielectric cables is required, a special type of vinyl jacket must be employed. Such a jacket has been developed and is now in service on these special cables.

To increase the mechanical protection of lines, particularly for naval use, an outer armor is sometimes employed. This is preferably of galvanized steel, but during the wartime emergency, aluminum wire has been substituted for the galvanized steel. Such armored cables are painted with an aluminum paint to fill the interstices in the armor and thus facilitate their watertight installation through bulkheads.

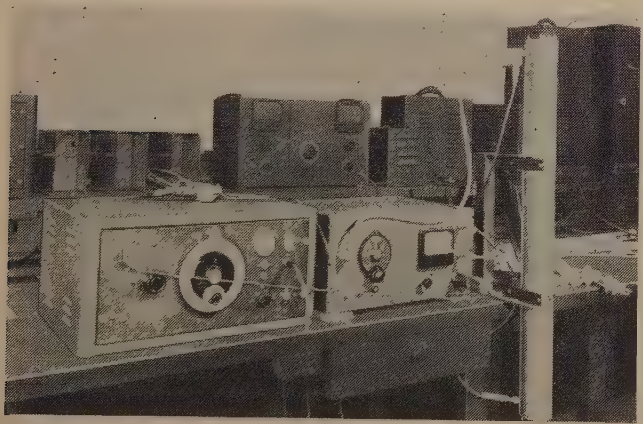

Fig. 4—Apparatus for measuring braid resistance.

Of great interest to the radio engineer is the contribution of the various components of the cable to the total measured loss or attenuation, and particularly, how this varies with frequency.

It is well known that the dielectric losses are a direct function of the electrical properties of the insulation and frequency of operation, and thus a measurement of the power factor and dielectric constant at those frequencies enables the contribution of the dielectric to the total loss to be calculated. Thus, taking polyethylene as an example: the power factor of polyethylene at 400 megacycles is 0.00030 and the corresponding dielectric constant is 2.27. The loss due to the dielectric is therefore $\alpha_{400} = 2.78 \times \sqrt{2.27} \times 0.00030 \times 400$ decibels per 100 feet $= 0.503$ decibel per 100 feet. Taking a standard cable, for example RG-8/U, the measured loss of this cable at 400 megacycles is 5.0 decibels per 100 feet. The contribution to the over-all loss by the center conductor and braid is therefore 4.5 decibels per 100 feet. To separate the loss due to the center conductor from that due to the braid, and to determine the effects of changing the lay of the braid on the coverage has been a somewhat complex problem and only empirical calculations have hitherto been made. Recently, however, Muller and Nordlin of the Federal Telephone and Radio Laboratories have developed an equipment (Fig. 4) which enables such measurements to be made, and the preliminary results are very valuable for future design work. At the present moment, due to lack of suitable oscillators, it is not possible to measure at frequencies other than 150 megacycles, but it is hoped shortly to have such oscillators and to be able to extend the usefulness of the equipment. Thus, taking the standard RG-8/U cable, which has a measured attenuation at 150 megacycles of 2.60 decibels per 100 feet, the contribution of the individual components of the cable is:

Dielectric component	= 0.189 decibel	per 100 feet
Center conductor	= 1.58 decibels	per 100 feet
Braid	= 0.903 decibel	per 100 feet
Total	2.67 decibels	per 100 feet.

It will be seen that the agreement is very good.

It is a well-known practical observation that the attentuation of a transmission line can be improved by increasing the lay of the braid wires; that is, by decreasing the number of braid-wire crossovers per inch. It has also been a matter of great interest, particularly from a raw-material conservation standpoint, to determine how the wire coverage, that is the ratio of surface of the dielectric covered by the braid wires to the total surface, affects the attentuation loss. Table II shows some of the results so far obtained. It will be seen that the loss due to the braid is markedly affected by the lay of the wires as would be expected, but that the per cent coverage is not a critical factor.

TABLE II

Lay angle in degrees	Loss in decibels per 100 feet	Coverage in per cent	Loss in decibels per 100 feet
63.4	1.94	97.75	0.888
55.9	1.36	93.30	0.856
47.7	1.22	86.70	0.825
38.6	1.05	77.90	0.804
26.1	0.888		

This particular experimental equipment will be of great use in studying the effect of different materials, such as silver-plated copper, and tinned copper versus plain copper wire, and also in studies to determine the effects of corrosion and/or flexing on the attenuation of transmission lines.

Testing Problems

The testing problems encountered in high-frequency solid-dielectric cables are of two types. First, those concerned with production testing, and second, specialized

problems concerned with the particular design and utilization of the cable type.

In general, factory production testing is chiefly a problem of the application of known techniques. It is necessary, however, to devise and build test equipments which are very rugged, quick acting, and as foolproof as possible. It must also be remembered that the equipments are to be operated by persons having little, if any, electrical background, and whose knowledge of high-frequency radio problems is substantially nil. The chief factory tests made are: capacitance, velocity of propagation, characteristic impedance, attenuation, dielectric strength, initial corona-starting voltage.

Capacitance is very conveniently measured at 1000 cycles and affords no special problem since standard equipment is employed.

The velocity of propagation is usually determined by measurement of the frequency at which a known length of cable, usually $\frac{3}{4}$ of a wavelength, resonates when terminated in an open circuit. Cables having a high attenuation cannot be measured by this method, because the resonance effects are not prominent enough to give an accurate determination. A convenient testing equipment, shown in Fig. 5, was developed by the Naval Research Laboratories. It consists of an oscillator, variable around 100 megacycles, coupled loosely to a noninductive loop attached to one end of the cable under test. This loop is tunable by a variable trimmer capacitor. Resonance is indicated by a dip in a grid meter. In the so-called "V-P Meter," the loop circuit, and the oscillator circuit are tracked and controlled by a single dial. When the cable under test is inserted into the meter, resonance occurs and is indicated at one position only on the dial which is calibrated to read velocity of propagation directly.

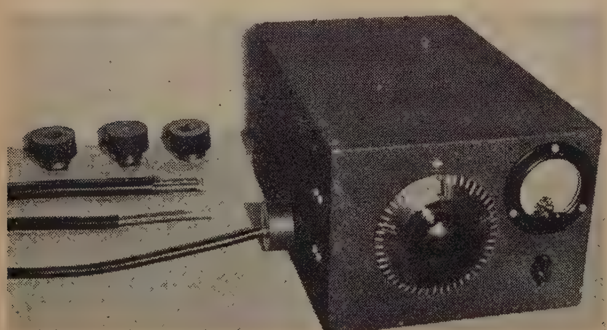

Fig. 5—Velocity-of-propagation meter.

The characteristic impedance is calculated from the measured capacitance and velocity of propagation by the usual formula. This calculation makes two assumptions which are justified: (1) That the effect of losses on characteristic impedance is negligible, which is true except for the lowest frequencies; (2) that the depth of penetration of current into the conductors is negligibly small, which is true for frequencies above 1 megacycle.

Fig. 6 shows the apparatus used by Federal Telephone and Radio Corporation for the measurement of attenuation at frequencies of 100, 200, 300, and 400 megacycles. The basic design was developed by the Naval Research Laboratories but was modified to give an equipment more suitable for production use and capable of higher precision. In this method, energy from a generator is fed directly into a calibrated vacuum-tube voltmeter and subsequently through the transmission line under test into the same voltmeter, the ratio of the two voltmeter readings giving the loss of the line. A diode voltmeter operated with a large input signal is employed to minimize transit-time effects and is calibrated at 60 cycles directly in decibels. The power output of the generator is adjusted with no transmission line connected until the voltmeter reads zero decibels. The generator is coupled to the input end of the line through a pair of critically coupled tuned circuits, and the output end of the line similarly coupled to the voltmeter, the critical coupling properly terminating the line in its characteristic impedance and eliminating reflection losses. The loss in the line can then be read directly in decibels. The spiral delay lines, or high-impedance lines, have relatively high attenuation values per unit physical length, and are

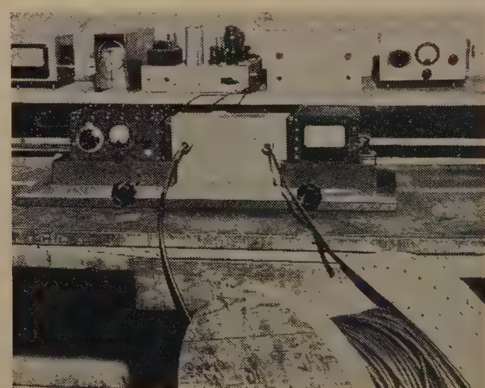

Fig. 6—Equipment for measuring the total attenuation of a transmission line.

therefore not capable of measurement on the above described equipments. An equipment, operating at 5 megacycles and measuring the resonant rise of voltage occurring when a sample of cable, an odd number of wavelengths long, is terminated in an open circuit, has been developed in our laboratories and has given very satisfactory operation. This method is based on the previous work of the Industry Service Division of R.C.A. Laboratories for use with television lines.

For cables designed for operation at high voltages, it is necessary to obtain as high an initial corona-starting potential as possible. Corona is a momentary discharge irregularly repeating at a rapid rate, and is caused by the ionization of air or other gaseous inclusions in the dielectric due to high electric-field strength. The corona-starting potential can be measured by a variety of means, but a convenient method comprises detecting the transient in the 60-cycle wave by means of an oscilloscope.

Special problems are met in the electrical testing of dual-coaxial balanced lines such as RG-23/U. Here problems of preparing lengths of cable which are electrically of the same length and which also show a high degree of electrical balance between the individual coaxials are often encountered. Fig. 7 shows a direct-reading electrical-length meter, developed by Federal Telephone and Radio Corporation, which operates on the principle that, at a fixed frequency, the input impedance of an open-circuited line is a function of the electrical length of the line. A voltage-regulated crystal-controlled generator applies a voltage to the input terminals of the line to be measured in series with a vacuum-tube radio-frequency milliammeter. The milliammeter is calibrated to read electrical length directly.

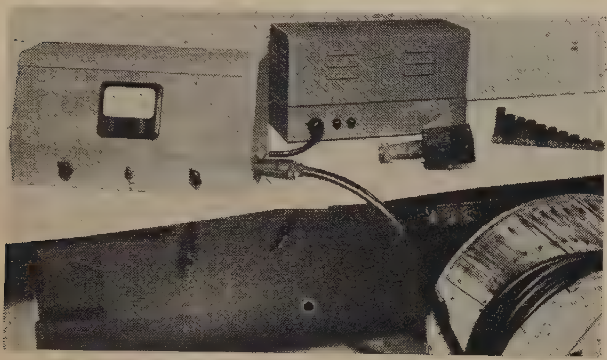

Fig. 7—Electrical-length meter.

To determine the electrical balance of dual-coaxial lines, particularly over a frequency range, a special balance test was developed. This set applies an equal voltage to both coaxials of the line from a motor-driven variable-frequency generator which slowly scans the frequency spectrum. At the far end of the line, a balanced-input amplifier operates a recording milliammeter to record the voltage unbalance.

A study of the graph recorded reveals the degree of unbalance and the frequency at which the greatest unbalance occurs. It will be seen that the development of test equipments has kept pace satisfactorily with the problems encountered in high-frequency transmission-line testing, although much work remains to be done to complete the picture. It will be appreciated that it is not possible at this stage to discuss other testing equipments which have been developed to meet special or specific problems, or to indicate the full extent of our knowledge, but the knowledge gained during this eventful period will be put to very good purpose in the years to come.

Future Outlook

A review of the current situation in the field of high-frequency solid-dielectric cables reveals the tremendous strides taken by a relatively inexperienced industry. The achievements of the past three years have been outstanding, but such successes must not prevent a critical examination of our present practices and theories, with an abandonment of those which fail to stand the test of scientific investigation. Certain fundamental weaknesses are already becoming apparent, and unless steps are taken to investigate these and to find solutions for them, what is now a virile section of the industry may stagnate and die, leaving other groups to find solutions to the problems along other lines.

In particular, continued research must take place into the field of dielectrics, with particular emphasis on the electrical characteristics and the operating temperature range. There are already many applications for transmission lines to operate at temperatures higher than 85 degrees centigrade, the present rating for polyethylene insulation, while the changes in characteristics observed over a temperature cycle due to the high coefficient of the thermal expansion of the same material are not very satisfactory. Braid designs, in view of recent work, can and should be modified to give the optimum mechanical and electrical properties.

One of the most important of the problems ahead is that of standardization. This must be considered not only from a national standpoint, but with an eye on international considerations, since we are now entering an era of widely expanded travel when equipments manufactured in one country will have to be serviced in another country. It will not be possible to uphold any nationalistic claims or demands in the face of a problem requiring over-all consideration, and the time is fast approaching when such problems must be tackled. A similar problem has been in the mind of industry for a number of years; namely, frequency allocation; and just as a series of conferences have been held on this subject, so must we have discussion and agreement on such problems as the impedance of cables, the maximum permissible attenuation at the various frequencies of operation, the over-all size of the cables and the connectors to be used therewith, etc. Without such standardization, the cable manufacturers will make whatever is called for by the customer and will have little inducement to study the problem deeply. The radio-equipment designer will continue to call for cables which may not constitute the best in the way of design, and the equipment installer and operator will go on decrying both. A vigorous discussion by all interested parties will clarify the position and enable the industry to go forward to even greater achievements.

The successful development of high-frequency solid-dielectric cables marks a very decided milestone in the history of radio, and it is up to the radio engineers to exploit more successfully the opportunity afforded them.


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HOTEL ASTOR—NEW YORK CITY
Headquarters for 1946 Winter Technical Meeting

1946 I.R.E. Winter Technical Meeting Program and Highlights
January 23, 24, 25, and 26, 1946
Hotel Astor, New York, N. Y.

Final plans have now been completed for what is confidently expected to be the most important I.R.E. Winter Technical Meeting in many years. When members gather from January 23 to 26 at the Hotel Astor for this first postwar meeting, they will be treated to an array of features and events that are among the most significant ever prepared for such an occasion.

Amid the crowded calendar of professional and social events, members will have at this gathering an unprecedented opportunity to orientate themselves in the postwar pattern of the electronics and radio fields, to gain an understanding of the Industry's reconversion program, and to catch up on the newest developments, and future prospects in the field.

Space in the Radio Engineering Show, a display of unprecedented variety and importance and four times the size of any former I.R.E. Radio Engineering Show, has been fully spoken for by more than 132 firms.

The total of 171 exhibits occupying two floors and foyer space in the Hotel Astor will represent a comprehensive cross section of the Industry's newest and most important postwar products and should provide mem-

bers with much information of value and interest to them in their particular fields. The most recent list of the firms which expect to take part in the Show is given on page 42 W. The list indicates how completely the suppliers of radio engineering equipment and services are cooperating to make this year's Show an outstanding success.

The annual I.R.E. Banquet will be held Thursday, January 24, 7:30 to 10:30 P.M. in the Grand Ballroom of the Hotel Astor and the President's Luncheon, honoring the Institute's incoming president, Dr. Frederick B. Llewellyn, to be held on Friday, January 25, at 12:30 P.M. in the Grand Ballroom. The principal speakers will be Dr. Frank B. Jewett, President of the National Academy of Sciences who has accepted the invitation to address the estimated 2500 guests at the Banquet, and Mr. Edgar Kobak, President of the Mutual Broadcasting System, who will act as Toastmaster; Mr. Paul A. Porter, Chairman of the Federal Communications System, who will be the speaker at the President's luncheon, and Mr. Ronald J. Rockwell, Engineering Director, Broadcasting Division, The Crosley Corporation, who will be master of ceremonies at the Luncheon.

At Thursday evening's Banquet, the two annual I.R.E. awards will be made: The Institute Medal of Honor given in recognition of distinguished service in radio communications; and the Morris Liebmann Memorial Prize, made to a member of the Institute who has made public during the recent past an important contribution to radio communications. In addition, 12 fellowships given by the Institute will be awarded.

Another enjoyable feature, the annual Cocktail Party, to be held Friday evening from 6:30 to 8:00 P.M. in the Grand Ballroom, promises to provide a pleasant medium for the renewing of old acquaintanceships and the making of new social and business contacts.

The splendid array of important technical papers on vital electronics and radio subjects will this year take on added significance with discussion of the many remarkable war developments and newly released information on hitherto restricted items.

The subjects of the papers give some hint of their importance. They include: Military Applications of Electronics; Frequency-Modulation and Standard Broadcasting; Circuits and Theory; Television; Radio Navigation Aids; Vacuum Tubes; Microwave Vacuum Tubes; Antennas; Radar; Microwave Technique; Industrial Electronics; Communication Systems and Relay Links; Radio Propagation; Broadcast Receivers; Quartz Crystals; and Crystal Rectifiers.

This year, as has been announced, The Institute of Radio Engineers will be host at a joint meeting with the American Institute of Electrical Engineers, scheduled to be held in the Engineering Society's auditorium on Wednesday evening, January 23. Major General Leslie R. Groves, Director of the Manhattan District, which is the code name for the atomic-bomb project, will speak on "Some Electrical Engineering and General Aspects of the Atomic-Bomb Project. To accommodate any overflow attendance such as occurred last year, arrangements have been made to install a public-address system and to reserve another large meeting room in the same building.

The women guests at this Meeting will be entertained with visits to the Museum of Costume Art and Sloane's House of Years followed by luncheon and an art exhibition at the Town Hall Club, and a Television Tour of Radio City. They will also be escorted on tours through the Cathedral of St. John the Divine and Riverside Church, as well as guests at luncheon.

The complete program of events for the three-day Meeting follows.

Greystone-Stoller

FRANK B. JEWETT

EDGAR KOBAK

PAUL PORTER

American

LEWIS M. CLEMENT

Gustafson

CATHEDRAL OF ST. JOHN THE DIVINE
Sanctuary and High Altar

RIVERSIDE CHURCH

Women's Program

(Tentative)

Thursday, January 24, 1946

11:00 A.M.–3:30 P.M.

Cathedral of St. John the Divine
Luncheon, Stoddards
Riverside Church

Friday, January 25, 1946

11:00 A.M.–4:00 P.M.

Museum of Costume Art—Sloane's House of Years
Luncheon and Art Exhibition, Town Hall Club
Television Tour of Radio City

FROM THE COLLECTION OF THE COSTUME INSTITUTE OF THE METROPOLITAN MUSEUM OF ART

PROGRAM

Wednesday, January 23, 1946

9:00 A.M.–5:30 P.M. Promenade	Registration and Sale of Tickets
9:30 A.M.–12:30 P.M. Coral Room	Annual Meeting of Sections' Representatives
12:30 P.M.–2:00 P.M. Rose Room	Luncheon for Sections' Representatives
2:00 P.M.–5:00 P.M. Coral Room	Annual Meeting of Sections' Representatives
4:00 P.M.–8:00 P.M. Eighth and Tenth Floors	Radio Engineering Show
8:00 P.M.–10:00 P.M. Engineering Societies Building	Joint Meeting of A.I.E.E. and I.R.E.

Thursday, January 24, 1946

8:30 A.M.–5:30 P.M. Promenade	Registration and Sale of Tickets
9:00 A.M.–7:00 P.M. Eighth and Tenth Floors	Radio Engineering Show
9:45 A.M.–10:30 A.M. Grand Ballroom	Annual Meeting of The Institute of Radio Engineers, Inc.

Technical Sessions

10:30 A.M.–12:30 P.M

Group A
Grand Ballroom
Military Applications of Electronics

Group C
Coral Room
Circuits and Theory

Group B
Rose Room
Frequency Modulation and Standard Broadcasting

Technical Sessions

2:00 P.M.–5:00 P.M.

Group A
Grand Ballroom
Television

Group C
Coral Room
Vacuum Tubes

Group B
Rose Room
Radio Navigation Aids

Thursday, January 24, 1946

Annual I.R.E. Banquet

Dress Optional

7:30 P.M.–10:30 P.M.

Grand Ballroom

Awarding of Medal of Honor, Morris Liebmann Memorial Prize, and Fellowship Awards

Address of Retiring President

Speaker: Dr. Frank B. Jewett, President of the National Academy of Sciences

Toastmaster: Mr. Edgar Kobak, President of the Mutual Broadcasting System, Inc.

Friday, January 25, 1946

9:00 A.M.–5:00 P.M. Promenade	Registration and Sale of Tickets
9:00 A.M.–10:00 P.M. Eighth and Tenth Floors	Radio Engineering Show

Technical Sessions

9:30 A.M.–12:00 NOON

Group A
Grand Ballroom
Microwave Vacuum Tubes

Group B
Rose Room
Antennas

Friday, January 25, 1946

President's Luncheon

Honoring President Frederick B. Llewellyn

12:30 P.M.

Grand Ballroom

Speaker: Mr. Paul Porter, Chairman, Federal Communications Commission

Master of Ceremonies: Mr. Ronald J. Rockwell, Engineering Director, Broadcasting Division, The Crosley Corporation

Friday, January 25, 1946

Technical Sessions

2:00 P.M.–5:30 P.M.

Group A
Grand Ballroom
Radar

Group C
Coral Room
Crystal Rectifiers

Group B
Rose Room
Microwave Technique

Friday, January 25, 1946

Cocktail Party

6:30 P.M.–8:00 P.M.

Grand Ballroom

Saturday, January 26, 1946

9:00 A.M.–3:00 P.M. Promenade	Registration
9:00 A.M.–2:00 P.M. Eighth and Tenth Floors	Radio Engineering Show

Technical Sessions

9:30 A.M.–12:00 NOON

Group A
Grand Ballroom
Industrial Electronics

Group C
Coral Room
Radio Propagation

Group B
Rose Room
Communication Systems and Relay Links

Radio Engineering Show—I.R.E. 1946 Winter Technical Meeting

Airadio, Inc.
Aircraft Marine Products, Inc.
Aircraft Radio Corporation
Aireon Manufacturing Corporation
Alden Products Company
Alpha Wire Corporation
Altec Lansing Corporation
American Brass Company
American Lava Corporation
American Phenolic Corporation
American Telephone and Telegraph Company
American Transformer Company
Amperex Electronic Corporation
Andrew Company

Ballantine Laboratories, Inc.
Alfred W. Barber Laboratories
Barker and Williamson
Bird Engineering Company
Boonton Radio Corporation
Brush Development Company
H. H. Buggie and Company
Burndy Engineering Company, Inc.

Allen D. Cardwell Manufacturing Corporation
Centralab, Division of Globe-Union, Inc.
Cherry Rivet Company
Sigmund Cohn and Company
Collins Radio Company
Communication Measurements Laboratory
Communication Products Company, Inc.
Communications
Condenser Products Company
Continental-Diamond Fibre Company
Cornell-Dubilier Electric Corporation
Corning Glass Works
Cornish Wire Company, Inc.
Crystal Research Laboratories, Inc.

Daven Company
DeMornay-Budd, Inc.
Tobe Deutschmann Corporation
Distillation Products, Inc.
John C. Dolph Company
Allen B. DuMont Laboratories, Inc.
DX Radio Products Company
Dynamic Air Engineering

Eastern Engineering Company
Eicor, Inc.
Eitel-McCullough, Inc.
Electrical Reactance Corporation
Electronic Laboratories. Inc.
Electronic Mechanics, Inc.
Electro-Voice, Inc.
Electronic Industries
Electronics
Erie Resistor Corporation

Fairchild Camera and Instrument Corporation
Fansteel Metallurgical Corporation
Federal Telephone and Radio Corporation
Ferris Instruments Company
F M and Television Magazine
A. W. Franklin Manufacturing Corporation

General Electric Company
General Electronics, Inc.
General Radio Company
Globe Wireless, Ltd.

Hallicrafters Company
Hammarlund Manufacturing Company, Inc.
Harco Tower, Inc.
Frederick Hart and Company, Inc.
Hewlett-Packard Company
Hytron Radio and Electronics Corporation

Industrial Instruments, Inc.
Industrial Products Company
Instrument Electronics
Instrument Specialities Company, Inc.
International Nickel Company, Inc.
International Resistance Company

J-B-T Instruments, Inc.
Jefferson-Travis Corporation
E. F. Johnson Company

Karp Metal Products Company, Inc.

Langevin Company, Inc.

Machlett Laboratories, Inc.
Madison Electrical Products Corporation
Maguire Industries, Inc.
Marion Electrical Instrument Company
Measurements Corporation
Mycalex Corporation of America

National Company, Inc.
National Research Corporation
National Union Radio Corporation
New York Transformer Company
North American Philips Company, Inc.

J. P. O'Donnell and Sons
Ohio Tool Company

Precision Tube Company
Presto Recording Corporation
Press Wireless, Inc.

Radio Corporation of America
Radio Craft
Radio Magazine
Radio News
Radio Receptor Company, Inc.
Raytheon Manufacturing Company
Rek-O-Kut Company
Remington-Rand Inc., Electronic Div.

Schweitzer Paper Company
Selenium Corporation of America
Shallcross Manufacturing Company
Sherron Electronics Company
Shure Brothers
Sola Electric Company
Solar Manufacturing Corporation
Sorensen and Company, Inc.
Sperry Gyroscope Company, Inc.
Sprague Electric Company
Stackpole Carbon Company
Standard Transformer Corporation
Star Expansion Products Company
Stupakoff Ceramic and Manufacturing Company
Superior Electric Company
Superior Tube Company
Sylvania Electric Products, Inc.

Telequip Radio Company
Television Magazine
Turney and Beale

U. S. Television Manufacturing Corporation
United Transformer Corporation

Ward Leonard Electric Company
Western Electric Company
Western Lithograph Company
Westinghouse Electric Corporation

Yardeny Engineering Company

Technical Sessions

2:00 P.M.–4:00 P.M.

Group A	**Group B**
Grand Ballroom	Rose Room
Broadcast Receivers	Quartz Crystals

Final Adjournment

4:00 P.M.

Committee Meetings

(Open to Members of Committees Only)

Wednesday, January 23, 1946

MORNING

Antennas
Frequency Modulation
Radio Receivers
Radio Wave Propagation

AFTERNOON

Circuits
Membership
Railway and Vehicular Communications
Research
Television
Vacuum Tubes

Thursday, January 24, 1946

MORNING

Education
Standards

AFTERNOON

Public Relations

Institute News and Radio Notes

Board of Directors

November 7 Meeting: At the regular meeting of the Board of Directors, which was held on November 7, 1945, the following were present: W. L. Everitt, president; G. W. Bailey, executive secretary; S. L. Bailey, W. L. Barrow, E. F. Carter, R. F. Guy, R. A. Heising, treasurer; Keith Henney, F. B. Llewellyn, Haraden Pratt, secretary; B. E. Shackelford, W. O. Swinyard, H. M. Turner, H. A. Wheeler, L. P. Wheeler, and W. C. White.

Approval of Executive Committee Actions: The actions of the Executive Committee taken at its October 3, 1945, meeting were unanimouly approved.

Annual Meeting of the Board: The Annual Meeting of the Board of Directors will be held on January 9, 1946.

Elections

The report of the Tellers Committee was accepted and the following nominees declared elected:

President—1946
F. B. Llewellyn

Vice-President—1946
E. M. Deloraine

Directors—1946–1948
W. R. G. Baker V. M. Graham
D. B. Sinclair

Committee

The following were selected as members of the Appointments Committee:

F. B. Llewellyn, *Chairman*

S. L. Bailey	L. C. F. Horle
W. L. Everitt	B. E. Shackelford
Keith Henney	D. B. Sinclair

Duplicate Publication of Papers: The Board of Directors established the following policy in connection with duplicate publication of papers:

"While in general it is the policy not to publish papers which have appeared elsewhere; nevertheless, in certain cases of outstanding importance, that policy shall not be considered obligatory."

Dr. Alfred N. Goldsmith: The Board extended expressions of sympathy in the illness of Dr. Goldsmith with best wishes for a speedy recovery and a return to his duties.

Browder J. Thompson

Memorial Fund: A check for $4000 for the Browder J. Thompson Memorial Fund was presented and an account of the status of the Memorial program outlined.

Award: The Browder J. Thompson Award was established in the manner and under the specifications outlined in the letter from Dr. R. R. Law to Mr. Pratt dated October 29, 1945, part of which is as follows:

"This award shall be known as the Browder J. Thompson Memorial Prize. Its purpose shall be to stimulate research in the field of radio and electronics and to provide incentive for the careful preparation of papers describing such research. The award shall be made annually to the author or joint authors under thirty years of age at date of submission of original manuscript (in case of joint authrship, all authors shall be under thirty years of age at date of submission of original manuscript) for that paper of sound merit recently published in the technical publications of The Institute of Radio Engineers which, in the opinion of the Awards Committee of the Institute, constitutes the best combination of technical contribution to the field of radio and electronics and presentation of the subject."

RTPB: It was unanimously approved that The Institute of Radio Engineers contribute toward the expenses of the Radio Technical Planning Board.

Co-operation and Liaison between I.R.E. and Foreign Societies: It was unanimously approved that the Board request the Executive Secretaries of the Institution of Electrical Engineers, London, England, and the Société Française des Radioelectriciens, Paris, France, to co-operate in publishing in the Proceedings of the I.R.E. and Waves and Electrons and both foreign publications statements inviting visitors from engineering societies abroad to get in touch with I.R.E., the British Society, and the Société Française, and that further co-operation between the societies and the I.R.E. be carried out.

Executive Committee

November 7 Meeting: The Executive Committee Meeting, held on November 7, 1945, was attended by W. L. Everitt, president; G. W. Bailey, executive secretary; S. L. Bailey, W. L. Barrow, E. F. Carter, R. A. Heising, treasurer; and Haraden Pratt, secretary.

Membership: Approval was given to the 371 applications for membership in the Institute as listed on page 38A of the December, 1945, issue of the Proceedings. These applications are as follows:

For Transfer to Senior Member Grade	25
For Admission to Senior Member Grade	12
For Transfer to Member Grade	76
For Admission to Member Grade	90
For Admission to Associate Grade	129
For Admission to Student Grade	39
	371

Editorial Department: Because of the temporary incapacity of the Editor to carry out his responsibilities, the Technical Editor, Mr. R. D. Rettenmeyer, was assigned to the special duty of Director of the Editorial Staff, reporting to the Executive Secretary.

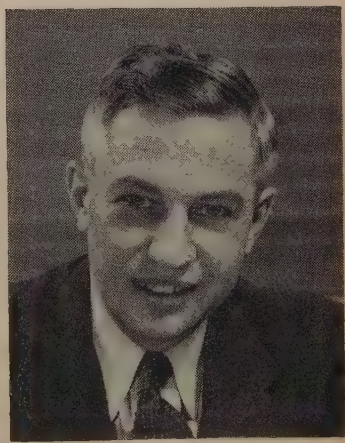
Yearbook: The Yearbook will be sold to nonmembers for $5.00.

Broadcast Engineering Conference: The Institute of Radio Engineers will continue cooperation with The Ohio State University and the University of Illinois and the National Association of Broadcasters in the Broadcast Engineering Conference which is to be held at the Ohio State University in Columbus, Ohio, during the week of March 18 to 23, 1946.

I.R.E.-RMA Committee: Raymond F. Guy, chairman of the Standards Committee, was invited as a representative of the I.R.E. to attend the meeting of the Executive Committee of the RMA Engineering Department which was held in Rochester on November 12, 1945. The recommendations of the I.R.E.-RMA Co-ordination Committee which was chiefly concerned with joint standards were accepted and approved.

Rochester Fall Meeting

Over one thousand registrants at the Rochester Fall Meeting, held on November 12 and 13, discussed their individual and industry problems and certain wartime and postwar developments. The Fall Meeting Committee awarded a plaque for distinguished service to Dr. Lee Du Bridge, director of the Radiation Laboratory, for the able manner in which he administered the vast National Defense Research Committee project which led to radar and our widespread and effective use of it during the war. At the banquet, Dr. Du Bridge outlined the capabilities and possibilities of the use of radar in a war-free world, especially in its service to navigational problems and to the safe flight landing of airplanes in all kinds of weather conditions.



Lee A. DuBridge

The following papers were delivered to capacity audiences of those in attendance at the 1945 Meeting:

"A Coaxial Modification of the Butterfly Circuit," by E. E. Gross of General Radio Company, contained considerable information on the mechanical problems arising in

constructing measuring equipment to operate in the thousand-megacycle regions.

"The Radio Proximity Fuze," by H. Trotter, Jr., of the Eastman Kodak Company, and "Proximity-Fuze Tubes," by Marcus A. Acheson of Sylvania Electric Products, Inc., not only disclosed a great deal of data on the use of these remarkable wartime devices but went to considerable length into the construction of the miniature tubes that made the fuzes possible.

"Microwave Radar," by Donald G. Fink of *Electronics*, brought engineers up to the minute on material that has been released for discussion. This included descriptions, including illustrations, of the apparatus for applying radar on frequencies as high as 10,000 megacycles and interesting facts about the actual usage to which radar was put during the war.

"High-Quality Sound Recording on Magnetic Wire," by L. C. Holmes of Stromberg-Carlson Company, gave those present a revelation of the advances that have been made in extending the frequency range and decreasing the distortion in wire recording. Demonstrations from RCA's new "unbreakable" vinylite records indicated not only the low record noise and wide dynamic range but also the fact that there is now very little difference between disk and wire recording, provided both are done correctly.

"The Aurora and Geomagnetism," by C. W. Gartlein of Cornell University, entertained the audience with many of the still unresolved perplexities of the aurora and of the effects of magnetic storms on communication. The lecture was illustrated with many photographs, including some in color, of aurora displays.

"Recent Developments on Converter Tubes," by W. A. Harris and R. F. Dunn of the Radio Corporation of America, discussed a new oscillator mixer good at 100 megacycles and above and thus applicable to the new frequency-modulation receivers. This tube has sufficiently high transconductance to be useful at the frequency indicated and sufficient conversion transconductance to operate at a 5- to 10-microvolt input level.

"War Influence on Acoustic Trends," by Hugh S. Knowles of the Jensen Radio Manufacturing Company, revealed much of the wartime usage of acoustic equipment and materials for morale (plus and minus), for training purposes, for getting speech and other forms of intelligence into and out of regions of high ambient-noise levels plus descriptions of "Bull" horns, "squawk" boxes, and other acoustic devices.

"Germanium Crystals," by Edward Cornelius of Sylvania Electric Products, Inc., gave many details of the wartime renaissance of crystal detectors indicating that the long-dormant crystal was henceforth a device to be reckoned with, due to modern engineering.

Television was treated by D. B. Smith of Philco Corporation in his paper "Industry Standardization Work in Television" and by E. W. Engstrom of the Radio Corporation of America who presented "A Review of the Technical Status." Under the title "Comments on Existing Television Systems from the Measurement Viewpoint," Jerry Minter of Measurements Corporation proposed that the carrier of the television system be amplitude-modulated for the picture and that the same carrier be frequency-modulated for sound; that receiver intermediate-frequency stages be matched to the transmitter characteristics resulting in overemphasis of the low-frequency video signals and that the high frequencies be brought up to match the low frequencies by "postemphasis." These proposals evoked considerable discussion from the floor.

Not on the official program was a paper by C. W. Carnahan of the Zenith Radio Corporation giving the results of recent measurements on the comparative efficacy of the present frequency-modulation band and the new band (100 megacycles) at distances of 75 miles. A lively discussion over frequency-modulation allocations followed the presentation of the paper.

Joint Electron Tube Engineering Council

O. W. Pike (A'26–M'29–SM'43), chairman of the Joint Electron Tube Engineering Council, the newly formed agency of the National Electrical Manufacturers Association and the Radio Manufacturers Association, recently announced the complete organization of the Council which includes L. C. Hector (A'26–SM'43), A. Senauke (M'28–SM'43), G. R. Shaw (M'40–SM'43), and R. M. Wise (A'26–M'30–F'37). The Council approves standards before they are forwarded to NEMA and RMA and provides executive decisions as required. It also has the responsibility of guiding the seven committees established to deal with individual classes of tubes and the four committees established to co-ordinate such matters as sampling procedures, packaging, type designations, and mechanical standards.

Broad general policies and matters of financing the activities of JETEC are subject to approval of the Boards of RMA and NEMA handled by the directors of the Council who include W. R. G. Baker (A'19–F'28). Most of the work of JETEC during the past months has dealt with the needs of the Armed Services for the standardization of electron tubes necessary to the war. The Council, however, is primarily a peacetime organization and has been devoting some of its energy to postwar problems such as improved methods of defining tube types more accurately so that equipment may be designed with a better understanding of the problems of tube interchangeability.

Radio Industry's War Production

With a total war output of approximately $7\frac{1}{2}$ billion dollars, the radio industry produced nearly twice as much radio-radar communications equipment during the war than it produced radio equipment alone for civilian use in all the years since commercial radio began about 1922. This was recently revealed by the Radio Manufacturers Association which has just received new production records of the War Production Board Radio and Radar Division.

From January, 1942, until the war ended last summer, the radio industry's war production mounted to the huge total of $7,220,000,000, the records show. In addition, the industry produced about $250,000,000 in military equipment from September, 1941, until the end of that year, according to the Radio Manufacturers Association, bringing the aggregate contribution to the war effort to close to the $7\frac{1}{2}$ billion mark.

Best industry and trade statistics show that in the entire period of civilian radio beginning in 1922, the total volume of radio equipment manufactured was about $4,225,000,000, not including transmitting and communications equipment, the association announced. This is some $3\frac{1}{4}$ billion dollars less than the production total for war.

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Westman Amendment

The Westman Amendment, voting on an increase in membership dues, was passed by the members in August 1945.

Effective as of January 1, 1946, the new dues rate will be as follows:

Fellow	$10.00
Senior Member	10.00
Member	10.00
Associate	$10.00 and $7.00*
Student	3.00

* $7.00 for each year that is within the first five years of Associate membership, starting January 1, 1946. After five years the dues automatically shall be increased from $7.00 to $10.00. The five-year period is retroactive from January 1, 1946.

This constitutional amendment also discontinued the payment of all transfer fees and set the entrance fee at $3.00 for all membership grades with the exception of the Student grade which does not require an entrance fee.

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The Institution of Electrical Engineers (England)

The Institution of Electrical Engineers has offered I.R.E. members the privilege of subscribing for its named publications at the listed prices, which are half of the normal annual rates:

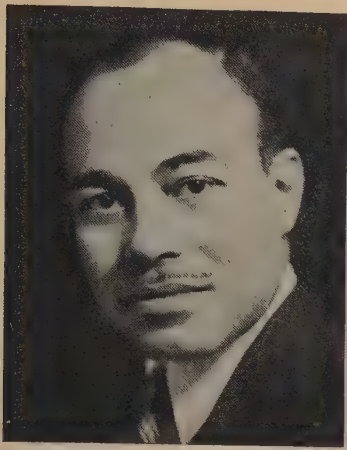
JOURNAL

Part I (General)	10s.6d. ($2)
Part II (Power Engineering)	15s.9d. ($3)
Part III (Radio and Communication Engineering)	10s.6d. ($3)
or	
all three Parts together	31s.6d. ($6) per annum.

SCIENCE ABSTRACTS

Section A (Physics Abstracts)	17s.6d. ($3.50)
Section B (Electrical Engineering Abstracts)	17s.6d. ($3.50)
or	
both Sections together	30s.0d. ($6)

A similar reduction (one half) on the Proceedings of the I.R.E. and Waves and Electrons in case of the IEE members will result in the special rate of $6.00 a year (which includes $1.00 foreign postage).



NELSON P. CASE

NELSON P. CASE

Nelson P. Case (A'26–M'31–SM'43) has joined the staff of the Hallicrafters Company, Chicago, Illinois, as chief engineer of the receiver division.

In 1924, Mr. Case was graduated from Stanford University with an A.B. degree in physics, and in 1926 he received an E.E. degree. He became active in geophysical work and held the position of assistant physicist at the Bureau of Standards, Washington, D. C., in 1928. The following year, Mr. Case was named research physicist in the department of engineering research at the University of Michigan. In 1930, he became a member of the staff of Hazeltine Electronics Corporation where he was engaged in various capacities for thirteen years, later having charge of the New York license laboratory of that organization. For the past two years, Mr. Case has been director of engineering design and development for the Hamilton Radio Corporation in New York City.

Mr. Case, holder of approximately thirty patents on radio-receiver circuits, is vice-chairman of the committee on broadcast and short-wave home receivers of the Radio Manufacturers Association's engineering department. He also serves on various other committees of this organization; namely, television receivers, systems, very-high-frequency receivers, and the executive committee of the engineering department's receiver section. Mr. Case is a member of Panel 6—Television Panel—of the Radio Technical Planning Board, a Fellow of the Radio Club of America, and a member of Phi Beta Kappa and Sigma Xi.

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E. FINLEY CARTER

E. Finley Carter (A'23–F'36) recently was elected a vice president in charge of industrial relations of Sylvania Electric Products, Inc. Previously he was chief radio engineer and later, as director of industrial relations he set up the company's industrial relations department. He will continue to be responsible for this function.

After he was graduated from Rice Institute in 1922, Mr. Carter joined the General Electric Company as a development engineer in the development of three early high-power transmitters, WGY in Schenectady, KGO in Oakland, and KOA in Denver. He was division engineer in charge of the special development division, handling television and the facsimile development program. In May, 1929, he became director of the engineering division of United Research Corporation, a subsidiary of Warner Pictures, designing radios, circuits, and receivers.

Mr. Carter was a director of The Institute of Radio Engineers in 1945 and is an Associate member of the American Institute of Electrical Engineers.

E. FINLEY CARTER

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HERBERT J. REICH

Herbert J. Reich (A'26–M'41–SM'43), educator and author of scientific articles and textbooks, has been appointed professor of electrical engineering at Yale University, his work starting on January 1, 1946.

Dr. Reich was born on Staten Island, New York, on October 25, 1900. He received the M.E. degree from Cornell University in 1924, and the Ph.D. degree in physics in 1929. Since that time he has been on the staff of the University of Illinois, where he was professor of electrical engineering. On January 1, 1944, he was granted a leave of absence to join the staff of the radio research laboratory at Harvard University.

He has specialized in the field of electron tubes and electron-tube circuits, and has published approximately forty papers on these and related subjects in technical periodicals. He is author of "Theory and Applications of Electron Tubes," "Principles of Electron Tubes," and co-author of "Ultra-High-Frequency Techniques."

Professor Reich is a Senior Member of the Institute of Radio Engineers. He has

HERBERT J. REICH

served on the Board of Editors and several committees. During 1944, he was a Member of the Board of Directors of The Institute of Radio Engineers. He is a member of the American Institute of Electrical Engineers, the American Physical Society, the American Association for the Advancement of Science, and the Society for the Promotion of Engineering Education.

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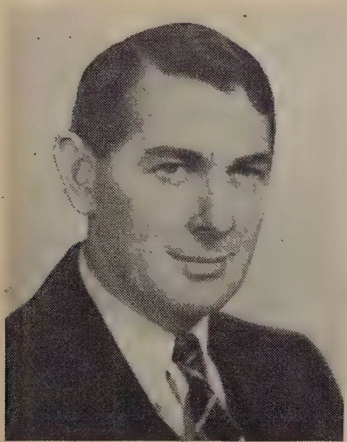
ROBERT J. GLEASON APPOINTED COMMUNICATIONS SUPERINTENDENT

Robert J. Gleason (A'31–M'39–SM'43) recently was appointed communications superintendent of the Pacific-Alaska Division, Pan American World Airways, with headquarters in San Francisco.

Mr. Gleason, who served with the U.S. Army since September, 1942, has been released from active service, in which he was a Lieutenant Colonel. During the past three years he served successively in Alaska, in the Aleutians, where he was in charge of all airways communications and was directly responsible for the installation of all radio communications in the Aleutian chain, in India and China, where since 1944 he was in charge of the 69th Army airways communications system and the 63rd AACS group in Kunming, China. These radio ground facilities served the 14th Air Forces under Major General Claire Chennault and the Indo-China Wing of the ATC. All "over the hump" flying between India and China was dependent on the Kunming communications center.

Originally joining Pan American in 1932, Mr. Gleason helped pioneer developments in Alaska first as chief operator in Fairbanks and later as communications superintendent for the company's operations throughout that area. He supervised the installation of communications facilities from Fairbanks to Juneau as well as fourteen of the radio stations throughout Alaska and Canada.

I.R.E. People



Clure H. Owen

Clure H. Owen

Clure H. Owen (A'32) joined the general engineering department of the American Broadcasting Company on October 1, 1945, as allocations engineer.

He will study allocations problems for standard-broadcast, frequency-modulation, and television facilities; be responsible for the design of directional-antenna systems; determine the location of suitable transmitter sites; and generally work towards the improvement of network coverage. He also will co-operate with the station relations department of the American Broadcasting Company in advising affiliates regarding allocation problems. Prior to joining the American Broadcasting Company, Mr. Owen was with the Federal Communications Commission as assistant chief of the broadcast-engineering division.

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Lloyd C. Sigmon

Major Lloyd C. Sigmon (A'29) has been appointed radio communications officer for the U. S. Group Control Council at Berlin. In recognition of his services in the European Theater of Operations, Major Sigmon was awarded the Legion of Merit Medal, and was made an honorary member of the French Signal Corps for his assistance to that organization. He has also recently been given the Order of the British Empire.

In prior years, Major Sigmon attended the school of electrical engineering, Milwaukee, Wisconsin, and from 1935 to 1940, was chief engineer for KCMO, Kansas City, Missouri. He then held the position of director of engineering for KMPC, Los Angeles, California, until his entry into the Armed Services as Captain in the U. S. Army Signal Corps in 1943. Shortly thereafter, he became chief radio engineer officer for communications in the European Theater. Before June of 1944, Major Sigmon selected the sites and directed the installation and operation of high-frequency and very-high-frequency radio stations in the United Kingdom for transatlantic and cross-channel communications in support of military operations on and after D Day. This included the planning for and implementation of the system which carried, throughout the world, the first news of the Allied invasion of the Continent.

Major Sigmon became engaged in the provision of radio communications facilities for Headquarters, European Theater of Operations, in Paris, where twelve high-power, high-frequency radio transmitting and receiving equipments were made operational within a month after that city's liberation. Upon completion of this project, he began the building of high-power mobile

Lloyd C. Sigmon

radioteletype stations to provide rapidly installed communications in support of the armies advancing in Germany. Among these is the 60-kilowatt, multichannel, suppressed-carrier, radioteletype, and radiotelephone station, whose equipment in actual operations was serviceable within twelve hours in Germany, providing the first army communications from that country to the United States.

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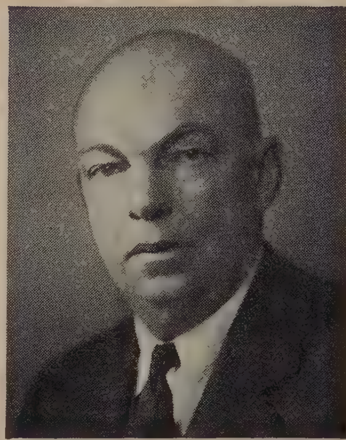
Major Armstrong Talks on Frequency Modulation

"Frequency Modulation is now coming into its rightful place in the communication field," said Major Edwin H. Armstrong (A'14–F'27) in an address delivered before the Cedar Rapids Section of The Institute of Radio Engineers on October 24.

Major Armstrong sketched the history of frequency modulation and compared it with amplitude modulation in regard to noise and distortion. Using charts to illustrate his talk, he said that noise and distortion are inherent in both the transmission and reception of amplitude-modulation signals, and that if pre-emphasis were used to



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Edwin H. Armstrong

bring up the high audio response, distortion in amplitude-modulation is aggravated. He pointed out that the use of "exalted carrier" in an amplitude-modulation receiver greatly improves the quality of reception.

"Distortion due to multipath transmission is practically nonexistent in frequency modulation," said Armstrong. He recounted his extensive investigation into that phenomenon, and his conclusion that multipath transmission would cause distortion only in mountainous country, and then only in extremely small areas.

The Major predicted that an 80-mile radius could be served from one frequency-modulation station without the aid of booster transmitters. The variety of programs transmitted and the number of different stations required for a given locality will be about the same as the existing amplitude-modulation service.

The frequency-modulation receivers will have to be of very high quality and will be relatively expensive if they are to be capable of reproducing frequency-modulation transmissions with the high quality that is possible with frequency-modulation.

Discussing the inception of frequency modulation, Major Armstrong said that he started looking for a static eliminator back about 1914, and that he worked a little longer than most people did. He then hit upon the idea of frequency shift-keying, and from that went into frequency modulation.

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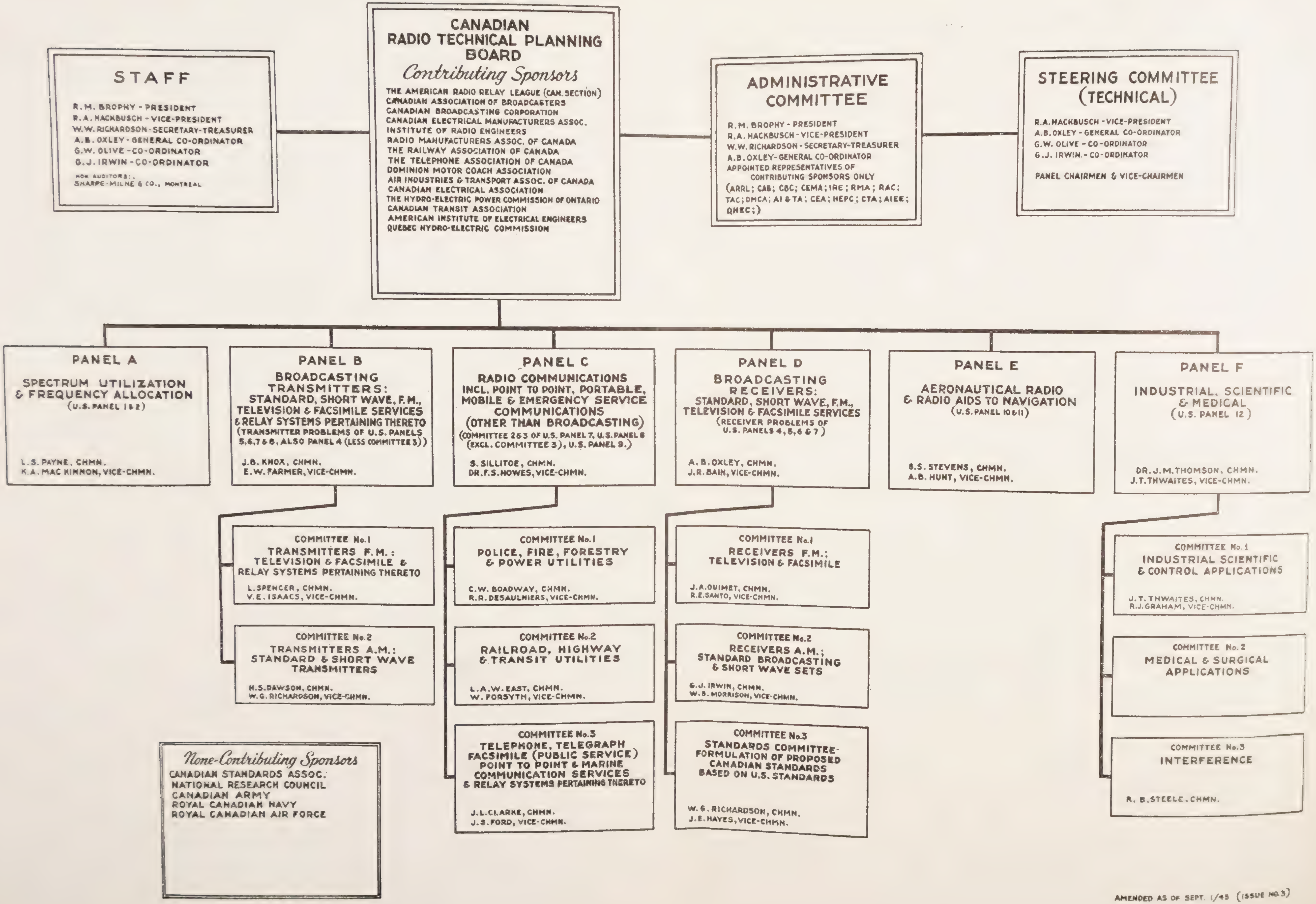
John J. Guarrera

John J. Guarrera (S'42–A'44), formerly a staff member of the Radiation Laboratory, Massachusetts Institute of Technology, is now affiliated with the engineering staff of Bernard Rice's Sons, Inc., New York, which during the war produced a wide range of electronic devices. It will continue to develop, engineer, and manufacture in the microwave, ultra-high-frequency, and radio-frequency fields.

Map, showing new territorial boundaries of sections, effective January 1, 1946.

ORGANIZATION CHART



AMENDED AS OF SEPT. 1/45 (ISSUE NO. 3)

SECTIONS

Chairman		Secretary
R. A. Holbrook 146 Lawrenceville Rd. Decatur, Ga.	ATLANTA January 18	I. M. Miles 554 14 St., N.W. Atlanta, Ga.
R. N. Harmon 1920 South Rd. Mt. Washington, Baltimore, Md.	BALTIMORE	F. W. Fischer 714 S. Beechfield Ave. Baltimore, Md.
C. C. Harris Tropical Radio Telegraph Co. Box 584, Hingham, Mass.	BOSTON	A. G. Bosquet General Radio Co. 275 Massachusetts Ave. Cambridge 39, Mass.
A. DiMarco Carabobo 105 Buenos Aires, Argentina	BUENOS AIRES	H. Krahenbuhl Transradio Internacional San Martin 379 Buenos Aires, Argentina
J. M. Van Baalen 282 Orchard Dr. Buffalo 17, N. Y.	BUFFALO-NIAGARA January 16	H. W. Staderman 264 Loring Ave. Buffalo, N. Y.
F. M. Davis Collins Radio Co. 855 35 St., N.E. Cedar Rapids, Iowa	CEDAR RAPIDS	J. A. Green Collins Radio Co. 855 35 St., N.E. Cedar Rapids, Iowa
Cullen Moore 327 Potomac Ave. Lombard, Ill.	CHICAGO January 18	L. E. Packard General Radio Co. 920 S. Michigan Ave. Chicago 5, Ill.
J. D. Reid Box 67 Cincinnati 31, Ohio	CINCINNATI January 15 Engineering Society Headquarters "Sound and Hearing" John D. Reid	P. J. Konkle 5524 Hamilton Ave. Cincinnati 24, Ohio
H. B. Okeson 4362 W. 58 St. Cleveland 9, Ohio	CLEVELAND January 24	A. J. Kres 16911 Valleyview Ave. Cleveland 11, Ohio
H. W. Sundius Southern New England Telephone Co. New Haven, Conn.	CONNECTICUT VALLEY January 17 HARTFORD "The Motorola Walki-Talkie, Handi-Talkie and the New 152–160 Megacycle Portable Mobile Equipment" Daniel E. Noble	L. A. Reilly 989 Roosevelt Ave. Springfield, Mass.
J. D. Mathis 811 Telephone Bldg. Dallas, Texas	DALLAS-FT. WORTH	B. B. Honeycutt 9025 Roanoak Dallas 18, Texas
L. B. Hallman 3 Crescent Blvd. Southern Hills Dayton, Ohio	DAYTON January 16 Dayton Engineers' Club "Electronic Aspects of the Geiger-Mueller Counter Tube and Spectrometer" Josef Heyd and Dexter H. Reynolds	Joseph General 411 E. Bruce Ave. Dayton 5, Ohio
L. H. Larime Technical Service, Inc. 693 Monroe St. Detroit 26, Mich.	DETROIT January 18	R. R. Barnes 1810 Sycamore Royal Oak, Mich.
N. L. Kiser Sylvania Electric Products, Inc. Emporium, Pa.	EMPORIUM February 7 "Sound Recording on Magnetic Wire" L. C. Holmes	D. J. Knowles Sylvania Electric Products, Inc. Emporium, Pa.
H. I. Metz Civil Aeronautics Authority Experimental Station Indianapolis, Ind.	INDIANAPOLIS	V. A. Bernier 5211 E. 10 Indianapolis, Ind.
R. N. White 4800 Jefferson St. Kansas City, Mo.	KANSAS CITY	Mrs. G. L. Curtis 6003 El Monte Mission, Kansas
B. S. Graham Sparton of Canada, Ltd. London, Ont., Canada	LONDON, ONTARIO	C. H. Langford Langford Radio Co. 246 Dundas St. London, Ont., Canada

Books

Elementary Electric-Circuit Theory, by Richard H. Frazier

Published (1945) by McGraw-Hill Book Co., Inc., 330 W. 42 St., New York 18, N. Y. 424 pages+10-page index+ix pages. 267 illustrations. $5\frac{1}{2}\times 8\frac{1}{2}$ inches. Price, $4.00.

This book treats elementary electric-circuit theory on a general broad basis which takes advantage of the present-day overlapping of the electric-power, communications, and electronics fields. The problems, illustrative examples, and figures are carefully selected from all of these fields to give the student the broad base so important today. The book is designed to provide the technical foundation in circuit analysis in electrical-engineering courses irrespective of the expected branches of specialization.

A knowledge of physics and mathematics usually acquired by the end of the sophomore year is sufficient for successful use of this text. The author claims no great glee in going out of his way to use a sophisticated mathematical approach, nor does he cramp the development of subjects by avoiding mathematics. A familiarity with the concepts of the differential and integral calculus and a mastery of algebra and plane trigonometry are prerequisites.

The book opens with a chapter on electric-circuit definitions and concepts which establishes a common starting point in such things as the frequency spectrum, fields and circuits, circuit elements, symbols, and terminology. Resistance networks are then treated because their simplicity allows concentration on principles. Chapters on basic alternating-current concepts and complex algebra lay the foundation for the chapter on impedance networks. Nonsinusoidal waves and polyphase networks are then covered in separate chapters. The last chapter deals with the classical method of transient analysis. Abbreviated tables in the appendix supply sufficient data for the problems found at the end of each chapter. A welcome feature rarely found in such texts is reference to historical sources such as the basic works of Ampere, Joule, Ohm, Coulomb and others.

The author has done a commendable job in arrangement and exposition, especially from the pedagogical standpoint.

F. ALTON EVEREST
Moody Institute of Science
947 Stanford St., Santa Monica, Calif

Chairman		Secretary
Fred Ireland General Radio Co. 1000 N. Steward St. Hollywood, Calif.	Los Angeles January 15	Walter Kenworth 1427 Lafayette St. San Gabriel, Calif.
L. A. W. East Canadian Pacific Railway 204 Hospital St. Montreal, Que., Canada	Montreal, Quebec February 13	R. R. Desaulniers Canadian Marconi Co. Box 1690 (Place D'Armes) Montreal 1, Que., Canada
G. B. Hoadley 85 Livingston St. Brooklyn, N. Y.	New York February 6	J. T. Cimorelli RCA Victor Div. 415 S. Fifth St. Harrison, N. J.
W. A. Steel 298 Sherwood Dr. Ottawa, Ont., Canada	Ottawa, Ontario January 17	L. F. Millet 33 Regent St. Ottawa, Ont., Canada
D. B. Smith Philco Corporation Philadelphia 34, Pa.	Philadelphia	P. M. Craig Philco Corporation Philadelphia 34, Pa.
J. A. Hutcheson 852 N. Meadowcroft Ave. Pittsburgh 16, Pa.	Pittsburgh January 14 Mellon Institute, 8 P.M. "Magnetic-Tape Recording" L. A. Umbach "Working with Microwaves" R. G. Plaisted and S. A. Leiner	C. W. Gilbert 52 Hathaway Ct. Pittsburgh 21, Pa.
Kenneth Johnson 5212 N.E. 73 St. Portland, Ore.	Portland	C. W. Lund R.F.D. 4, Box 858 Portland, Ore.
G. R. Town Stromberg-Carlson Co. Rochester 3, N. Y.	Rochester January 17	A. E. Newlon Stromberg-Carlson Co. Rochester 3, N. Y.
B. B. Miller 2356 A Lawrence St. St. Louis 10, Mo.	St. Louis	N. J. Zehr 1538 Bradford Ave. St. Louis 14, Mo.
David Kalbfell 941 Rosecrans San Diego, Calif.	San Diego	Clyde Tirrell U. S. Navy Radio and Sound Laboratory San Diego 52, Calif.
David Packard Hewlett-Packard Co. Palo Alto, Calif.	San Francisco	William Barclay 955 N. California Ave. Palo Alto, Calif.
E. H. Smith 823 E. 78 St. Seattle 5, Wash.	Seattle February 14	W. R. Hill University of Washington Seattle 5, Wash.
F. H. R. Pounsett Research Enterprises, Ltd. Leaside, Ont., Canada	Toronto, Ontario	Alexander Bow 137 Oxford St. Guelph, Ont., Canada
H. E. Hartig University of Minnesota Minneapolis, Minn.	Twin Cities	M. R. Ludwig 315 E. 24 St. Minneapolis, Minn.
H. A. Burroughs Munsey Bldg. Washington 4, D. C.	Washington January 14	L. C. Smeby 4801 Connecticut Ave., N.W. Washington 8, D. C.
Harry Smithgall Sylvania Electric Products, Inc. Plant No. 1 Williamsport, Pa.	Williamsport February 6	F. L. Burroughs 2030 Reed St. Williamsport 39, Pa.

SUBSECTIONS

R. C. Higgy 2032 Indianola Ave. Columbus, Ohio	Columbus February 8	Warren Bauer 376 Crestview Rd. Columbus 2, Ohio
P. B. Laeser 9410 Harding Rd. Milwaukee, Wisc.	Milwaukee	E. L. Cordes 3304 N. Oakland Ave. Milwaukee, Wisc.
L. J. Giacelleto 9 Villa Pl. Eatontown, N. J.	Monmouth	C. D. Samuelson 5 Russel Ave. Fort Monmouth, N. J.
W. C. Johnson Princeton University Princeton, N. J.	Princeton	J. G. Barry Princeton University Princeton, N. J.
H. E. Ellithorn 417 Parkovash Ave. South Bend 17, Ind.	South Bend January 17	J. E. Willson WHOT, St. Joseph and Monroe Sts. South Bend, Ind.

Proceedings of the National Electronics Conference

Published (1945) by the National Electronics Conference, Inc. c/o Illinois Institute of Technology, 1951 W. Madison St., Chicago 12, Illinois. 605 pages+x pages. 389 illustrations, 9×6 inches. Price $3.00.

To the 2000 persons who attended the first National Electronics Conference in Chicago in 1944, this book needs no introduction and little comment. It is a straightforward and carefully edited presentation of fifty-one papers presented at the conference, with brief abstracts of five others. Edited by Messrs. Dudley, Beam, and Foster, of the Conference Publications Committee, the book is divided into sections as follows: General Papers, Electron-Tube Developments, Electronic Measurements and Controls, Communication Electronics, Industrial Electronics, Electronic Aids to Medical Science, and Theoretical Electronics and Design.

It is a tribute to those who organized the Conference that such an array of material was made available, contributed by so many leading figures of the art. The material is up to date as of October, 1944, and the majority portion of it has lasting reference value. The advantage of publication in book form is manifest, not only for reference, but also to permit leisurely perusal of papers heard as well as papers missed because it was physically impossible to attend all the sessions. It is highly recommended.

Donald G. Fink
McGraw-Hill Publishing Co.
New York 18, N. Y.

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Section Territory Assignment

Early in 1945, the Board of Directors adopted a new policy concerning Section territory in the Uuited States. It was decided that instead of assigning a small territorial area around the city center of each section, the remaining unassigned territory of the United States would be assigned to sections. It was also planned to do the same with Canada.

The Sections' Committee met and drew up a suggested assignment. The assignment for the United States has been approved and is to take effect January 1, 1946. The proposed assignment for Canada has not yet been approved as the desires of the members and sections in that area have not been received.

The new method of assigning territory means that every member of the Institute in the United States becomes a member of some section. As such he will be on the membership list of his section and should receive notices of meetings.

The assignments were made on the basis of distance and travel habits so far as they were known to the committee. The proposed division was submitted to the Chairmen of all Sections with a request for comments. Several suggestions for changes were made and where there was some doubt about the

wisdom of the changes, polls were taken of the members residing in the areas in question. These areas have now been assigned in accordance with the majority vote received from such members. The attached map, which shows the outlines of the territories for the sectons was drawn up and approved.

The assignment of all territory to sections will call for certain new operational policies upon the formation of new sections. Heretofore, the formation of most sections required only the assignment of unassigned area to the proposed new section. A few new sections have been formed which called for removing some area from an older section. Now all new sections will of necessity be carved out of present sections. However, the opportunity for the establishment of new sections will not be reduced at all. The practice followed in such establishment will be modified only to the extent of asking the new and old section officers to agree on their respective areas. If any difference in opinion exists, the Executive Committee will poll the members in any disputed area, or decide the matter upon whatever pertinent information is available.

The assignment of all territory among sections is a prelude to a proposed new arrangement for naming members of the Board of Directors. An effort is being made to have more members on the Board from sections remote from New York and plans are being formulated which it is thought will accomplish this result. Such plans envisage grouping of sections into a few large areas called "Regions" and providing these regions with a co-ordinating council and representation on the Board. In the course of time, therefore, after the necessary constitutional changes have been made, a new type of territorial division will be superimposed upon the sectional division. It is planned that each region will be composed of a number of sections with their territories. The number of these regions, and their boundaries will be determined by the Board of Directors. In the meantime, the establishment of the new section territories will have gone into effect for a period of time that it is hoped will be sufficiently long to turn up any shortcomings that should be corrected.

It is suggested that all members refer to the map, determine how they are affected, and register any objections before the plans have been carried so far as to involve serious complications in making any changes. Suggestions for changes should be addressed to the Institute Office along with reasons therefor. A member requesting a change should give the name of his county, as the smallest division made in area is along county lines.

Canadian Radio Technical Planning Board

Of interest to the membership of The Institute of Radio Engineers is the Organizational Chart of the Canadian Radio Technical Planning Board, as shown on the attached insert. The Canadian Council of the I.R.E. is one of the sponsor organizations of the Canadian RTPB. It is expected that the new Board will contribute constructively to the development of radio and electronics in Canada and presumably will collaborate effectively with the corresponding groups in the United States.

Notice to Sections

Transfers and Admissions to Higher Grade Membership

In the interest of simplifying and expediting transfers and admissions to the grades of Member and Senior Member in the Institute, a procedure has been established whereby the Section Membership Committees co-ordinate the routing of reference forms on behalf of applicants residing in the areas of the respective Sections. This procedure was outlined in President Everitt's letter of July 11, 1945, to Section Chairmen and Secretaries, and its substance is here repeated for the information of the Sections and their members.

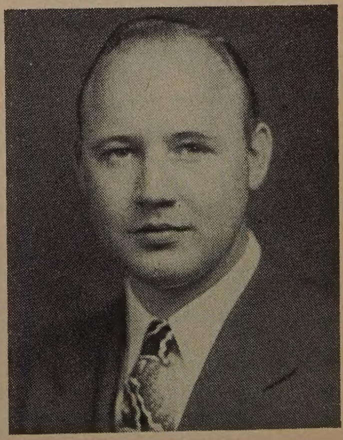
An applicant for transfer or admission to higher grade is required to fill in an application form in which he will designate persons who will act as references. The Section Secretary or member of the Section Membership Committee who may receive such application will then forward to each of the designated references the appropriate reference form and small white envelope on which shall be inscribed the applicant's name in the space provided for this purpose. A supply of these forms and envelopes has been sent to Section Secretaries. The reference, upon completing the form, shall seal it in the white envelope, bearing his own name and address in the upper left-hand corner, and return it to that official of the Section from whom it was received. In order to maintain the confidential character of the information on the reference form, these envelopes will *not* be opened by the Sections, but, when all the responses have been received from the references pertaining to any one applicant, they shall be enclosed together with the application in the large white envelope supplied and forwarded to the I.R.E. Headquarters in New York.

Since in the past much of the delay in effecting transfers or admissions to higher membership has resulted from the length of time often required to obtain responses from references, it is felt that the handling of this phase of the work locally by the several Sections should greatly expedite the process. The establishment of this procedure is not intended in any way to preclude the former practice in which an applicant sends an application form directly to the I.R.E. Headquarters. In this case, the Headquarters office will continue to send out the necessary forms to the indicated references.

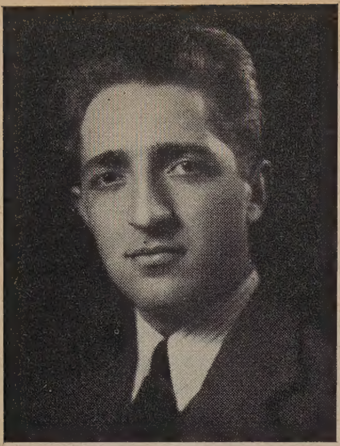
Contributors to Waves and Electrons

Andrew Alford

Andrew Alford (A'35–M'40–F'42) was born on August 5, 1904, at Samara, Russia. In 1924 he was graduated from the University of California, and from 1925 to 1927 he was a university Fellow and graduate student there. During 1927 and 1928, Mr. Alford was a teaching Fellow in physics at the California Institute of Technology. He was a research engineer with the Fox Film Corporation, West Coast division, from 1929 to 1931; a geophysical prospecting and consulting engineer from 1931 to 1934; an engineer with the Mackay Radio and Telegraph Company from 1934 to 1941; head of the air-navigation laboratory of the Federal Telephone and Radio Corporation; and head of the direction-finder and antenna division, Radio Research Laboratory, Harvard University, from 1943 to date.



Robert E. Graham



Armig G. Kandoian

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Robert E. Graham (M'45) was born at Kansas City, Missouri, on October 4, 1916. He received the B.S. degree in electrical engineering from Purdue University in 1937. In the same year Mr. Graham joined the technical staff of the Bell Telephone Laboratories, where he has since been engaged in television research. During this period he has engaged in part-time graduate work at Columbia University.

He is an associate member of Sigma Xi and the Optical Society of America.

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Armig G. Kandoian (S'35–A'36–SM'44) was born at Van, Armenia, on November 28, 1911. He received the B.S. degree in 1934, and the M.S. degree in electrical communication engineering in 1935, from Harvard University.

Mr. Kandoian became associated with the International Telephone and Telegraph Corporation in 1935, and has remained with that organization to date, working primarily on very-high-frequency and ultra-high-frequency developments dealing with antennas, radiation, measurements, link communication, and air navigation. Since 1941 he has been in the laboratories division of the Federal Telephone and Radio Corporation.

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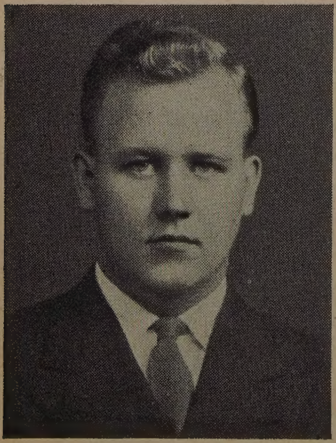

Frank J. Lundberg

He is a member of Tau Beta Pi, Harvard Engineering Society, and the American Institute of Electrical Engineers.

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Frank J. Lundburg (S'38–A'40) was born on December 28, 1916, at Ashton, Idaho. He received the B.S. degree in electrical engineering in 1940 from Purdue University, and from 1940 to 1941 he was a student engineer in the general engineering department of the Columbia Broadcasting System.

From 1941 to 1942 Mr. Lundburg worked in the short-wave division of the same department, and from 1942 to date he has been associated with the Federal Telephone and Radio Laboratories. Since 1943 he has been doing graduate study at Stevens Institute of Technology.

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F. W. Reynolds was born at Colton, New York, on February 10, 1897. He received the B.S. degree in electrical engineering from Union College in 1919, and the Ph.D. degree in physics from Cornell University in 1924. He was an instructor at Union College in 1919 and 1920, and at Cornell University from 1920 to 1924. In 1924 he joined the department of development and research of the American Telephone and Telegraph Company. Since 1934 he has been a member of the technical staff of the Bell Telephone Laboratories. Dr. Reynolds has been associated with the development work in the wire transmission of pictures and in television.

F. W. Reynolds

He is a member of the Optical Society of America, the American Physical Society, and Sigma Xi.

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Helen M. Stote was born in Colorado Springs, Colorado. She received the A.B. degree in English from Stanford University and the M.A. degree in Spanish from the University of Wisconsin. In 1929, she became an assistant to the Editorial, Advertising, and Circulation Departments of the Institute of Radio Engineers. In 1930, Miss Stote was made assistant editor, in 1943, associate editor, and in 1945, publication manager.

Helen M. Stote

Also, she has served as chairman of the Women's Committee for New York Conventions since 1937.

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Chester B. Watts, Jr. (A'41) was born at Washington, D.C., on June 16, 1918. He received the B.S. degree in 1940 from the Massachusetts Institute of Technology. From 1940 to 1942 he was associated with the radio navigation laboratory of the Federal Telephone and Radio Laboratories, and since 1942 he has been at the Aircraft Radio Laboratory, Wright Field, Dayton, Ohio.

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A. J. Warner was born on March 25, 1913, at London, England. He received the B.Sc. degree in chemistry from the University of London in 1935. He joined the staff of Standard Telephones and Cables as a metallurgical chemist in 1930, obtained a leave of absence in 1932, and returned in 1935 to the same organization, where he worked until 1940 as raw-material chemist and as research chemist on problems relating to electrical insulation.

In 1940, Mr. Warner was transferred to the New Insulants factory at Enfield, Middlesex, and in 1941, to the International Standard Electric Company. He is at present employed by the Federal Telephone and Radio Corporation, Newark, New Jersey, as technical director of the Intelin Division of the Company.

A. J. Warner